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COMPARISON OF FOUR SEQUENTIAL PROBABILITY RATIO TESTS.

Sung Hwan Wie

NAVAL POSTGRADUATE SCHOOL

Monterey, California

THESIS

COMPARISON OF FOUR SEQUENTIAL PROBABILITY RATIO TESTS

by

Sung Hwan Wie

March 1978

Thesis Advisor: Donald R. Barr

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Operating characteristics, E[N], V[N] and error rates for these tests are presented.

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COMPARISON OF FOUR SEQUENTIAL PROBABILITY RATIO TESTS

by

Sung Hwan Wie
Lieutenant, Republic of Korea Navy
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Submitted in partial fulfillment of the
requirements for the degree of

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I. OBJECTIVE

In this thesis the Sequential Rayleigh Test and three sequential binomial tests, which are applicable to testing bombing system accuracy, are compared by computer simulation.

The objective of this thesis is to investigate expected sample size, variance of sample size, error rates of these tests, and to investigate their robustness.

II. DESCRIPTION OF THE TESTS

A. INTRODUCTION

There are two types of tests of a system (which is taken to be a bombing system in what follows). The sample size is determined as a direct result of the experiment in one case (and is therefore random), and the sample size is selected prior to commencing the test in the other case.

The former procedure is called a sequential test [1]. The test procedures being considered here are sequential, and are based on assumed circular normal distribution (that is, bivariate normal) with the same variance in each coordinate. In the coordinate system of the target plane it is assumed that:

$$X \sim \text{Normal } (0, \sigma^2)$$

$$Y \sim \text{Normal } (0, \sigma^2).$$

The origin of coordinate system is the target, and X and Y are distances from the weapon impact to the target along the X and Y axes.

If it is further assumed that X and Y are independent, then $\frac{X}{\sigma}$ is normal with mean 0, variance 1. $\frac{Y}{\sigma}$ is normal with mean 0, variance 1. Thus $(\frac{X}{\sigma})^2 + (\frac{Y}{\sigma})^2$ has Chi-square distribution with two degrees of freedom.

The density function of Chi-square with two degrees of freedom is [2].

$$f_T(t) = \frac{1}{2 \cdot \Gamma(1)} \cdot e^{-t/2}, \quad t>o$$

$$= \frac{1}{2} e^{-t/2}$$

This is the exponential density function with parameter $\lambda = \frac{1}{2}$. Here,

$\frac{X^2}{\sigma^2} + \frac{Y^2}{\sigma^2} = \frac{X^2 + Y^2}{\sigma^2}$, where $X^2 + Y^2$ is squared miss distance of the impact from the target. Let this be another random variable Z. Then,

$$\frac{Z}{\sigma^2} \sim \text{Exp}(\tfrac{1}{2})$$

The density function of Z is derived as follows:

$$F_Z(z) = P[Z \leq z]$$

$$= P\left[\frac{Z}{\sigma^2} \leq \frac{z}{\sigma^2}\right].$$

Let $\frac{Z}{\sigma^2}$ be Z';

then $F_Z(z) = F_{Z'}\left(\frac{z}{\sigma^2}\right)$

Where Z' is exponentially distributed with parameter $\lambda = \frac{1}{2}$.

Then

$$F_{Z'}\left(\frac{z}{\sigma^2}\right) = 1-\exp[-z/2.\sigma^2] = F_Z(z).$$

From the relation that $\frac{Z}{\sigma^2} = Z'$ the density function of Z is [5]

$$\frac{1}{2\sigma^2}\exp[-z/2.\sigma^2] = f_z(z)$$

which is the exponential density function with parameter $\lambda = \frac{1}{2\sigma^2}$.

Assume C is the median of Z (C represents CEP^2 which is defined to be the median of the squared Circular Error Probable [6]).

Then by definition:

$$F_Z(C) = \frac{1}{2}$$

$$1 - \exp[-C/2.\sigma^2] = \frac{1}{2}$$

$$-\frac{C}{2\sigma^2} = \ln\left(\frac{1}{2}\right)$$

$$\sigma^2 = \frac{C}{2\ln 2}$$

Thus $Z \sim \mathrm{Exp}\left(\frac{\ln 2}{C}\right)$

If a bombing system has been specified to have median radial miss distance γ_0 and if a system with radial miss distance γ_1 is unacceptable, this can be tested with $C_0 = \gamma_0^2$ as the null hypothesized median and $C_1 = \gamma_1^2$ as the alternative hypothesized median. Here, $\frac{\ell n2}{C_0}$, $\frac{\ell n2}{C_1}$ are the parameters under null and alternative exponential distributions, respectively.

In what follows C_0 will be treated as median under null hypothesis and C_1 as median under alternative, where these quantities relate to the squared radial miss distribution.

B. TEST CASE A: SEQUENTIAL RAYLEIGH TEST

If a system is to be tested with null hypothesis H_o: $\text{CEP}^2 = C_o$ and alternative hypothesis H_1: $\text{CEP}^2 = C_1$ with type I and type II error rate α and β, then a sequential test for H_o against H_1 can be defined as follows. We shall call this the "Sequential Rayleigh Test" in what follows; it is an application of Wald's Sequential probability ratio test to the exponential situation described above.

Two positive constants are chosen, B and A, where B < A. At each stage (n-th experiment or observation), the probability ratio $\prod_{i=1}^{n} \frac{f(z_i;\, C_1)}{f(z_i;\, C_o)}$, called the likelihood ratio [3], is computed, where $f(z_i;\, C_j)$ is the exponential density function with parameter $\frac{\ell n 2}{C_j}$, $j = 1,2$

If $B < \prod_{i=1}^{n} \frac{f(z_i, C_1)}{f(z_i, C_o)} < A$, then another observation is made (this means the test enters the $(n+1)^{st}$ stage).

If the likelihood ratio does not fall in the interval (B,A), called the continuation region, the test terminates.

In termination, the conclusion is to:

$$\text{Accept } H_o \text{ if } \prod_{i=1}^{n} \frac{f(z_i, C_1)}{f(z_i, C_o)} \leq B;$$

$$\text{Reject } H_o \text{ if } \prod_{i=1}^{n} \frac{f(z_i, C_1)}{f(z_i, C_o)} \geq A.$$

For a test with approximate level of significance α and power $1-\beta$, one may define [1]

$$A = \frac{1-\beta}{\alpha}, \text{ and } B = \frac{\beta}{1-\alpha}.$$

Thus, an approximate bound for each stage can be obtained as follows:

The explicit form of the likelihood ratio is

$$\prod_{i=1}^{n} \frac{f(z_i, C_1)}{f(z_i, C_o)}$$

$$= \frac{(\ell n2/C_1).\exp[-z_1.\ell n2/C_1]. \text{ - - - } .(\ell n2/C_1).\exp[-z_n.\ell n2/C_1]}{(\ell n2/C_o).\exp[-z_1.\ell n2/C_o]. \text{ - - - } .(\ell n2/C_o).\exp[-z_n.\ell n2/C_o]}$$

$$= \frac{(\ell n2/C_1)^n. \, (\sum_{i=1}^{n} z_i).\exp[-\ell n2/C_1]}{(\ell n2/C_o)^n.(\sum_{i=1}^{n} z_i).\exp[-\ell n2/C_o]}$$

$$= \left(\frac{C_o}{C_1}\right)^n \cdot e^{-\left(\sum_{i=1}^{n} z_i\right) \cdot \left[\frac{\ell n 2}{C_1} - \frac{\ell n 2}{C_o}\right]}$$

This is equated to A and the logarithm is taken to obtain rejection bound R_n for the n-th stage;

$$\ell n\left(\frac{1-\beta}{\alpha}\right) = n . \ell n\left(\frac{C_o}{C_1}\right) - \left(\sum_{i=1}^{n} z_i\right) . \left[\frac{\ell n 2}{C_1} - \frac{\ell n 2}{C_o}\right]$$

The test rejects H_o if

$$\sum_{i=1}^{n} z_i \geq \frac{-\ell n\left(\frac{1-\beta}{\alpha}\right) + n . \ell n\left(\frac{C_o}{C_1}\right)}{(\ell n 2) . \left(\frac{1}{C_1} - \frac{1}{C_o}\right)} = R_n$$

where $\sum_{i=1}^{n} z_i$ is sum of squared radial miss distances.

The same procedure can be applied to B:

$$\ell n\left(\frac{\beta}{1-\alpha}\right) = n \ell n\left(\frac{C_o}{C_1}\right) - \left(\sum_{i=1}^{n} z_i\right) . \left(\frac{\ell n 2}{C_1} - \frac{\ell n 2}{C_o}\right)$$

The test accepts H_o if

$$\sum_{i=1}^{n} z_i \leq \frac{-\ell n\left(\frac{\beta}{1-\alpha}\right) + n \cdot \ell n\left(\frac{C_o}{C_1}\right)}{(\ell n 2) . \left(\frac{1}{C_1} - \frac{1}{C_o}\right)} = A_n$$

Here, it is assumed $C_1 > C_o$, which is, we envision, true in the bombing system test.

C. CASE B: SEQUENTIAL BINOMIAL TEST WITH $P_o = \frac{1}{2}$

In the study of bombing system the target is defined to be a point on the impact plane and impact of a bomb within (over) some distance $\sqrt{r}$ from the target is defined to be a hit (miss).

A null hypothesis that $\mathrm{CEP}^2 = C_o$ is to be tested against an alternative hypothesis that $\mathrm{CEP}^2 = C_1$, with Type I and Type II error rates α and β, where $C_o < C_1$.

Also this system can be tested with $\mathrm{CEP}^2 \le C_o$ as null hypothesis and $\mathrm{CEP}^2 \ge C_1$ as alternative hypothesis without altering the test procedure.

Let P_o be defined as the probability of hit under the null hypothesis, and P_1 to be the hit probability under the alternative hypothesis. Then under H_o, $\mathrm{CEP}^2 \le C_o$, which says the true median of squared radial miss distance is less than or equal to C_o, implies the probability of hit is greater than or equal to 0.5.

Similarly, $\mathrm{CEP}^2 \ge C_1$ implies the probability of hit is less than or equal to 0.5 under the alternative hypothesis. So the new hypotheses are defined as H_o; $P_o \ge 0.5$ and H_1: $P_1 \le 0.5$. From this hit or miss criterion r, a value of squared radial miss distance may be obtained which gives P_o value of 0.5; i.e., if

$$F_z(r) = 1-e^{-\frac{\ln 2}{C_o} \cdot r} = 0.5$$

then $\ln(0.5) = (\ln 2) \cdot (-\frac{r}{C_o})$, so $r = C_o$.

The alternate hit probability P_1 is found as follows:

$$P_1 = P_r \ [\text{hit} \mid CEP^2 = C_1]$$

$$= P_r \ [\text{observed squared radial miss distance} \leq r \mid CEP^2 = C_1]$$

$$= 1 - e^{-\frac{\ln 2}{C_1} \cdot r}$$

$$= 1 - e^{-\frac{\ln 2}{C_1} \cdot C_o} = 1 - 2^{-\frac{C_o}{C_1}}$$

Let the random variable Z_i be defined as:

$$z_i = \begin{cases} 0 \text{ if miss (squared miss distance} > r) \\ 1 \text{ if hit (squared miss distance} \leq r) \end{cases}$$

Then the likelihood ratio becomes

$$\prod_{i=1}^{n} \frac{f\,(z_i, P_1)}{f\,(z_i, P_o)}$$

$$= \frac{(P_1)^Z \ (1-P_1)^{n-Z}}{(1/2)^Z \ (1/2)^{n-Z}}$$

$$= \left(\frac{P_1}{1-P_1}\right)^Z \cdot (1-P_1)^n \cdot 2^n$$

Where $Z = \sum_{i=1}^{n} z_i$

For a sequential probability ratio test for the binomial situation, the experiment is continued as long as this value remains between B and A.

An approximate acceptance boundary is found by substituting $\frac{\beta}{1-\alpha}$ for B [1].

Then $\ln\left(\frac{\beta}{1-\alpha}\right) \geq \left(\sum_{i=1}^{n} z_i\right) \cdot \ln\left(\frac{P_1}{1-P_1}\right) + n \, . \ln(2(1-P_1))$.

Substituting $(1-2^{-C_o/C_1})$ for P_1 gives

$$\ln\left(\frac{\beta}{1-\alpha}\right) \geq \left(\sum_{i=1}^{n} z_i\right) \ln\left(\frac{1-2^{-C_o/C_1}}{2^{-C_o/C_1}}\right) + n . \ln \quad 2^{1-C_o/C_1}$$

Solving for $\sum_{i=1}^{n} z_i$, which is a convenient test statistic:

$$\sum_{i=1}^{n} z_i \geq \frac{\ln\left(\frac{\beta}{1-\alpha}\right) - n \cdot (1 - C_0/C_1) \cdot \ln 2}{\ln \quad (2^{C_o/C_1} - 1)} = A_n$$

(The inequality changes because $C_o < C_1$ implies $2^{C_0/C_1} - 1 \leq 0$.) Where $\sum_{i=1}^{n} z_i$ is the number of bombs which hit the target, out of the total fire, n. And substituting $\frac{1-\beta}{\alpha}$ for A, an approximate rejection boundary is found to be:

$$\sum_{i=1}^{n} Z_i \leq \frac{\ln\left(\frac{1-\beta}{\alpha}\right)-n.\left(1-\frac{C_0}{C_1}\right).\ln 2}{\ln\left(2^{C_0/C_1}-1\right)} = R_n$$

The test now operates as follows:

in stage n,

Accept H_0 if $\sum_{i=1}^{n} z_i \geq A_n$;

Reject H_0 if $\sum_{i=1}^{n} z_i \leq R_n$;

Continue to stage n+1 otherwise.

D. CASE C: A SEQUENTIAL BINOMIAL TEST WITH NULL PARAMETER WHICH MINIMIZES E[N] UNDER H_0.

This case also has null hypothesis $CEP^2 \leq C_0$ and alternative $CEP^2 \geq C_1$. Let $P_0(r)$ be the probability of hitting a target of radius $\sqrt{r}$, and let $P_1(r)$ denote that probability under H_1. In mathematical form:

$$P_0(r) = P_0(Z \leq r | CEP^2 = C_0) = 1-e^{-\frac{\ln 2}{C_0} \cdot r}$$

$$P_1(r) = P_r(Z \leq r | CEP^2 = C_1) = 1-e^{-\frac{\ln 2}{C_1} \cdot r}$$

Thus: $P_1(r) = 1 - [1 - P_0(r)]^{C_0/C_1}$

It is desired to determine r so as to minimize expected sample size n, required to test H_0 vs H_1 with Type I, II error rates α,β respectively, using the Binomial Sequential Probability Ratio test.

The average sample size function is

$$E\,(N) \cong \frac{(1-L(P))\,.\,\ell nA + L\,(P)\,.\,\ell nB}{P.\ell n\left(\frac{P_1(r)}{P_0(r)}\right) + (1-P).\ell n\left(\frac{1-P_1(r)}{1-P_0(r)}\right)}\quad,\ [1],\ [4]$$

where P is the true probability of hit and $L(P)=P_r[\text{accept } H_0|P]$.
If $P_0(r)$ is chosen to be the underlying hit probability, then $L(P_0(r))=1-\alpha$.

By substituting $L(P_0(r))=1-\alpha$,

$A = \frac{1-\beta}{\alpha}$, $P_1(r)=1-(1-P_0(r))\cdot C_0/C_1$, $B = \frac{\beta}{1-\alpha}$

in E[N] function, the expression above becomes

$$E(N) \cong \frac{\alpha\ell n\left(\frac{1-\beta}{\alpha}\right) + (1-\alpha)\cdot\ell n\left(\frac{\beta}{1-\alpha}\right)}{P_0(r)\cdot[\ell n\left(\frac{1-(1-P_0(r))^{C_0/C_1}}{P_0(r)}\right)] + (1-P_0(r))\ell n[(1-P_0(r))^{-1+C_0/C_1}]}$$

If a value of $P_0(r)$ which minimizes E(N) is obtained, r is also obtained from $P_0(r)$. The numerator is a negative constant as long as $\beta<0.5$ and $\alpha<0.5$ and the denominator is a negative variable depending on r.

Therefore, in order to minimize E(N) it is necessary only to maximize the absolute value of the denominator.

Let $\frac{C_o}{C_1} = \frac{1}{k}$

Then for various values of K, $P_o(r)$ may be found. Figure II-1 shows a plot of $P_o(r)$ vs k resulting from this minimization. As k approaches infinity $P_o(r)$ approaches 1.0; as k decreases to zero $P_o(r)$ decreases to around 0.63. Specifically, for k = 2

$$P_o(r) = 0.8416, \; P_1(r) = 0.6020.$$

From this r is found to be

$$- C_o \cdot \frac{\ell n(0.1584)}{\ell n\, 2} = 2.65836 \cdot C_o$$

Let Z be a random variable such that

$$z_i = \begin{cases} 0 & \text{if squared miss distance} > r \\ i & \text{if squared miss distance} \leq r \end{cases}$$

where r = 2.65836.

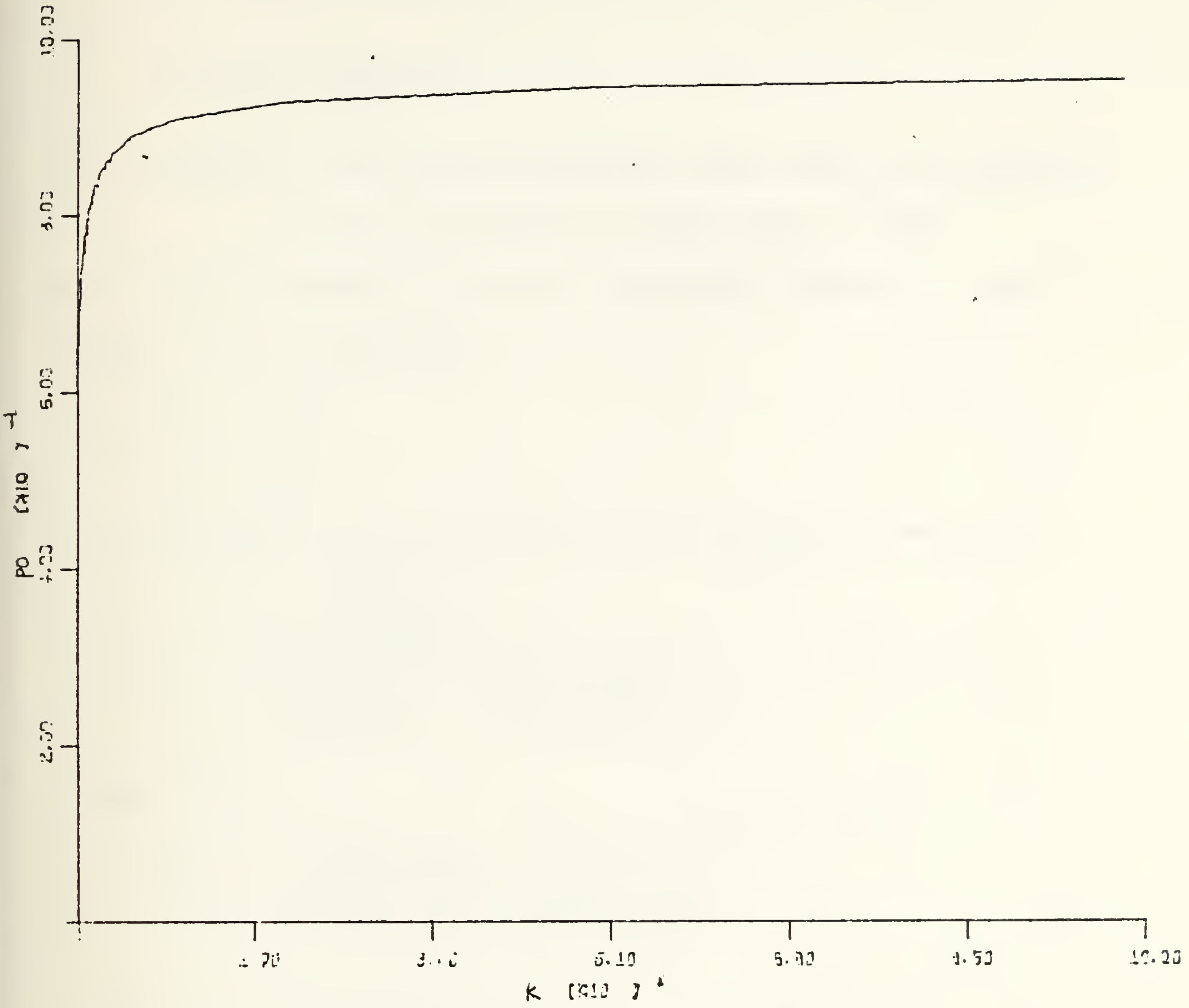
Then the hypotheses in this Binomial test becomes:

H_o: $P_o = 0.8416$, H_1: $P_1 = 0.602$.

If $P_o = 0.8416$, $P_1 = 0.602$ are substituted in the likelihood ratio (similar to Case B above), acceptance and rejection bounds at stage n are:

Accept H_o if

$$\sum_{i=1}^{n} z_i \geq -1.2564 \;.\; \ell n \left(\frac{\beta}{1-\alpha}\right) + n.0.7333 = A_n$$



K	0.001	0.1	0.3	0.5	0.90	1.01	1.414	1.5
P_o	0.632	0.655	0.707	0.745	0.789	0.800	0.820	0.824
K	2.0	3.0	5.0	7.0	10.0	20.0	50.0	100.0
P_o	0.842	0.863	0.886	0.899	0.910	0.928	0.944	0.953

FIGURE II-1

Reject H_o if

$$\sum_{i=1}^{n} Z_i \le - 1.2564 . \ln \left(\frac{1-\beta}{\alpha} \right) + n . 0.7333 = R_n$$

Continue otherwise.

E. CASE D: A Sequential Binomial test with null parameter which minimizes maximum value of E(N)

In the binomial situation described above, the hypotheses about the parameter are

H_o: $P = P_o$

H_1: $P = P_1$.

The maximum average value of E(N) occurs very nearly at

$$P' = \frac{\ln\left(\frac{1-P_o(r)}{1-P_1(r)}\right)}{\ln\left(\frac{P_1(r)}{P_o(r)}\right) - \ln\left(\frac{1-P_1(r)}{1-P_o(r)}\right)} \qquad [4]$$

from which

$$E(N) \cong \frac{\ln A . \ln B}{\ln\left(\frac{P_1(r)}{P_o(r)}\right) - \ln\left(\frac{1-P_1(r)}{1-P_o(r)}\right)}$$

$$= \frac{\ln A . \ln B}{\ln\left[\frac{1-(1-P_o(r))^{C_0/C_1}}{P_o(r)}\right] . \ln\left[\frac{(1-P_o(r))^{C_0/C_1}}{1-P_o(r)}\right]}$$

($P_1(r) = 1 - [1-P_o(r)]^{C_0/C_1}$, See Case B)

The numerator is a negative constant for given α and β.

Similarly as in Case C by maximizing the absolute value of the denominator, the maximum value of E[N] is minimized (or at least nearly so).

Figure II-2 shows the relation between $P_o(r)$ and $k = \frac{C_1}{C_o}$.

Specifically for k = 2

$P_o(r) = 0.89867$, $P_1(r) = 0.68167$ with $r = -\frac{\ell n(1-0.89867)}{\ell n 2} = 3.30286$ is obtained.

Hence the hypotheses for this case are H_o: P=0.89867, H_1: P=0.68167, and "hit" is defined by squared radial miss distance less than or equal to 3.30286.

Let Z be a random variable such that

$$Z_i = \begin{cases} 0 & \text{if squared miss distance} > 3.30286 \\ 1 & \text{if squared miss distance} \leq 3.30286. \end{cases}$$

Proceeding similarly as in Case B, the decision at stage n is

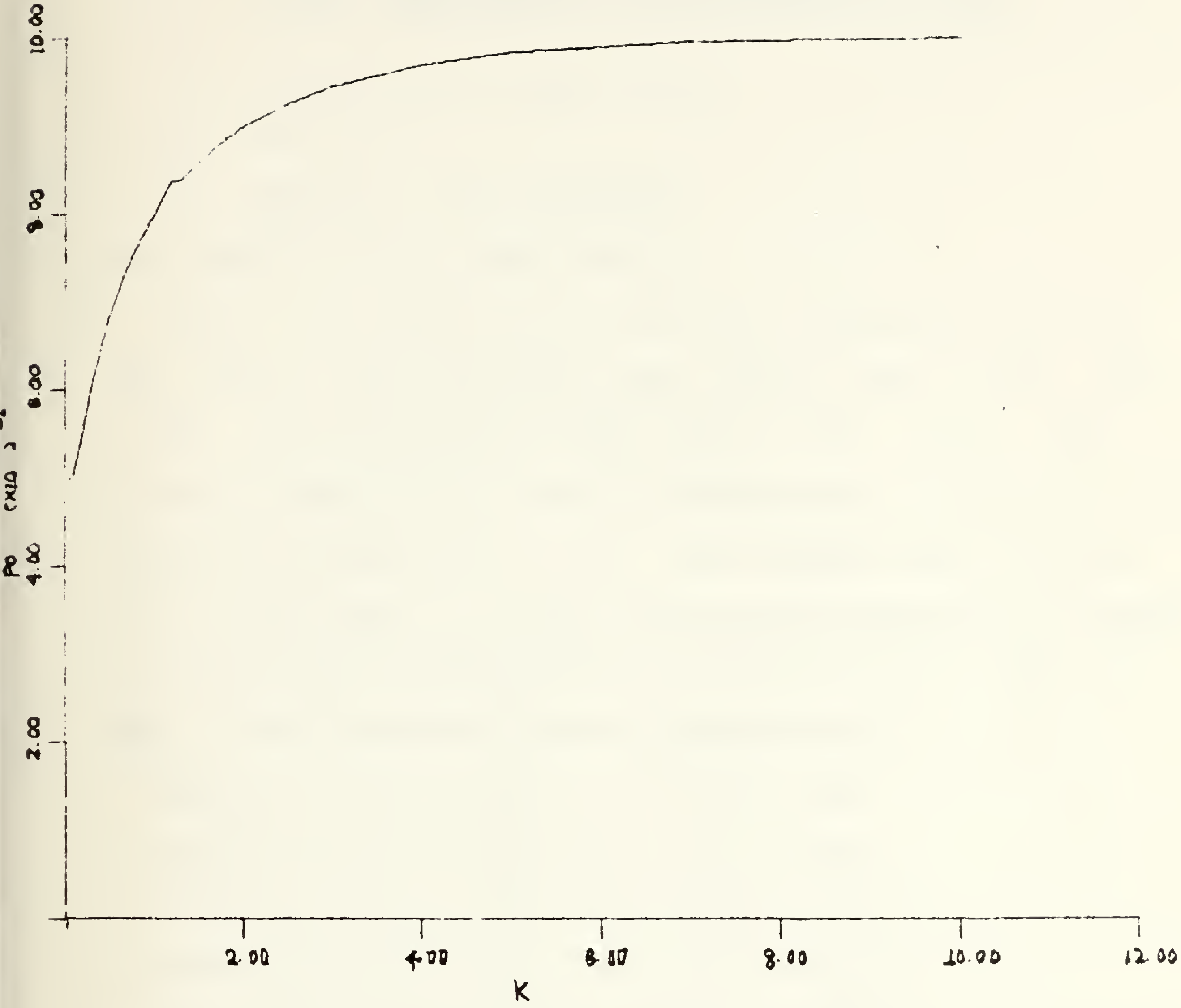
Reject H_o if

$$\sum_{i=1}^{n} Z_i \leq -1.42167 \,.\, \ell n\left(\frac{1-\beta}{\alpha}\right) + n.\ 0.80552 = R_n$$

Accept H_o if

$$\sum_{i=1}^{n} Z_i \geq -1.42167 \,.\, \ell n\left(\frac{\beta}{1-\alpha}\right) + n.\ 0.80552 = A_n$$

Continue to state n+1 otherwise.



K	0.05	0.1	0.3	0.5	0.7	0.9	1.01	1.414
P_o	0.5	0.504	0.602	0.681	0.738	0.780	0.794	0.851
K	1.5	2.0	2.5	3.0	4.0	5.0	7.0	10.0
P_o	0.859	0.899	0.925	0.944	0.968	0.982	0.994	0.999

FIGURE II-2

III. DETERMINATION OF SIMULATION FACTORS

For convenience the hypotheses

H_o: $CEP^2 = 1$

H_1: $CEP^2 = 2$ were selected.

This means k = 2 in Case C and D.

Type I and Type II error rates were selected to be 0.05. But actual simulation gives Type I error rates .0258, .0468, .0396, .0292 and Type II error rates .0396, .0402, .0420, .0400 for Case A, B, C, and D, respectively. This point is explained in Chapter 3.3 of [1]. Approximate error rates of 0.05, 0.05 (Type I, II) were obtained by adjusting the bounds A and B, which is possible by changing α, β in $A = \frac{1-\beta}{\alpha}$, $B = \frac{\beta}{1-\alpha}$. The adjustment factors used here are:

CASE A $\alpha = 2.0 \times 0.05$ $\beta = 1.2 \times 0.05$

CASE B $\alpha = 1.2 \times 0.05$ $\beta = 1.2 \times 0.05$

CASE C $\alpha = 1.5 \times 0.05$ $\beta = 1.15 \times 0.05$

CASE D $\alpha = 1.5 \times 0.05$ $\beta = 1.1 \times 0.05$

How many replications are enough? Assuming N_1, N_2 have Binomial distribution with probability of success 0.05, a sample size n is found such that $P_r(|\hat{P}_1-\hat{P}_2|<c) = 1-\alpha$, where α is the significance level, $\hat{P}_i$, i=1,2 is the estimation of P_i. The above equation implies

$$P_r(-c < \hat{P}_1 - \hat{P}_2 < c) = 1-\alpha$$

$$P_r(-c < \frac{N_1}{n} - \frac{N_2}{n} < c) = 1-\alpha$$

$$P_r(-nc < N_1 - N_2 < nc) = 1-\alpha$$

By the Normal approximation , $(N_1 - N_2) \overset{\cdot}{\sim}$ Normal with mean o and Variance 2np(1-p).

Thus

$$P_r \left[- \frac{nc}{\sqrt{2np(1-p)}} < \frac{N_1 - N_2}{\sqrt{2np(1-p)}} < \frac{nc}{\sqrt{2np(1-p)}} \right] = 1-\alpha$$

gives

$$\frac{nc}{\sqrt{2np(1-p)}} = Z_{1-\frac{\alpha}{2}}$$

The following table shows n for various values of α and c.

α	0.2	0.1	0.05	0.025	0.02
$Z_{1-\frac{\alpha}{2}}$ c	1.285	1.645	1.96	2.24	2.33
0.2	3.9	6.4	9.1	11.9	12.8
0.1	15.7	25.7	36.1	47.6	51.5
0.05	62.7	102.8	145.9	190.7	206.3
0.025	250.9	411.3	583.9	762.7	825.2
0.01	1568.7	2570.7	3649.5	4766.7	5157.5
0.001	156866.4	257072.3	361237.5	476672.0	515745.5

Assuming $c = 0.01$ and $\alpha = 0.02$, an approximate value of 5000 is obtained through the table. By this number of replications, obtaining a difference in estimated error rates greater than 0.01 is significant at level 0.02. Exponential random samples were generated by the Monte Carlo method. For an exponential variate T to have median m, it is necessary to use the scale parameter $\lambda = \frac{\ell n\, 2}{m}$

so $F_T(t) = 1 - e^{-\frac{\ell n 2}{m} \cdot t}$

But $U = F_T(T)$ is uniformly distributed on (0,1) [8].

Thus $2^{\frac{-T}{m}} = 1 - F(T) = 1 - U$ is also uniformly distributed.

Finally $\frac{T}{m} \cdot \ell n 2 = -\ell n\, U$, or

$$T = - m . \frac{\ell n\, U}{\ell n 2} .$$

By changing the median of the population sampled, the operating characteristic function, expected sample size at termination, and its variance can be estimated for each test, and those can be compared.

The medians to be generated are those which result in a 0.1 difference of operating characteristic function values in Case C, which is based on minimizing the maximum E(N), those which yield Type I, II error rates, three points which yield approximate maximum E(N) value (for Cases B, C, D), and five more points in both tails.

By changing the skewness of the sample distribution, comparison of the robustness of the four tests of hypotheses about the medians under null and alternative hypothesis was performed.

This is based on the assumption that the underlying distribution is WEIBULL with shape parameter α and scale parameter λ.

If $T \sim \text{WEIBULL}\{\alpha, \lambda\}$,

the distribution function of T is

$$F_T(T) = 1 - e^{-(\lambda T)^{\alpha}}$$

Let m be the median. Then

$$P_r[T \geq m] = P_r[T \leq m];$$

$$1 - e^{-(\lambda m)^{\alpha}} = 0.5,$$

$$\ell n(0.5) = -(\lambda \cdot m)^{\alpha} .$$

Solving for λ,

$$\lambda = \frac{1}{m}[\ell n 2]^{1/\alpha}$$

Now $1-e^{-(\lambda T)^{\alpha}} \sim U(o,1)$, so

$$\ell n(1-U) = -(\lambda T)^{\alpha} ,$$

$$T = \frac{1}{\lambda}[\ell n(U)]^{\frac{1}{\alpha}} , \text{ where } U \sim U(o,1).$$

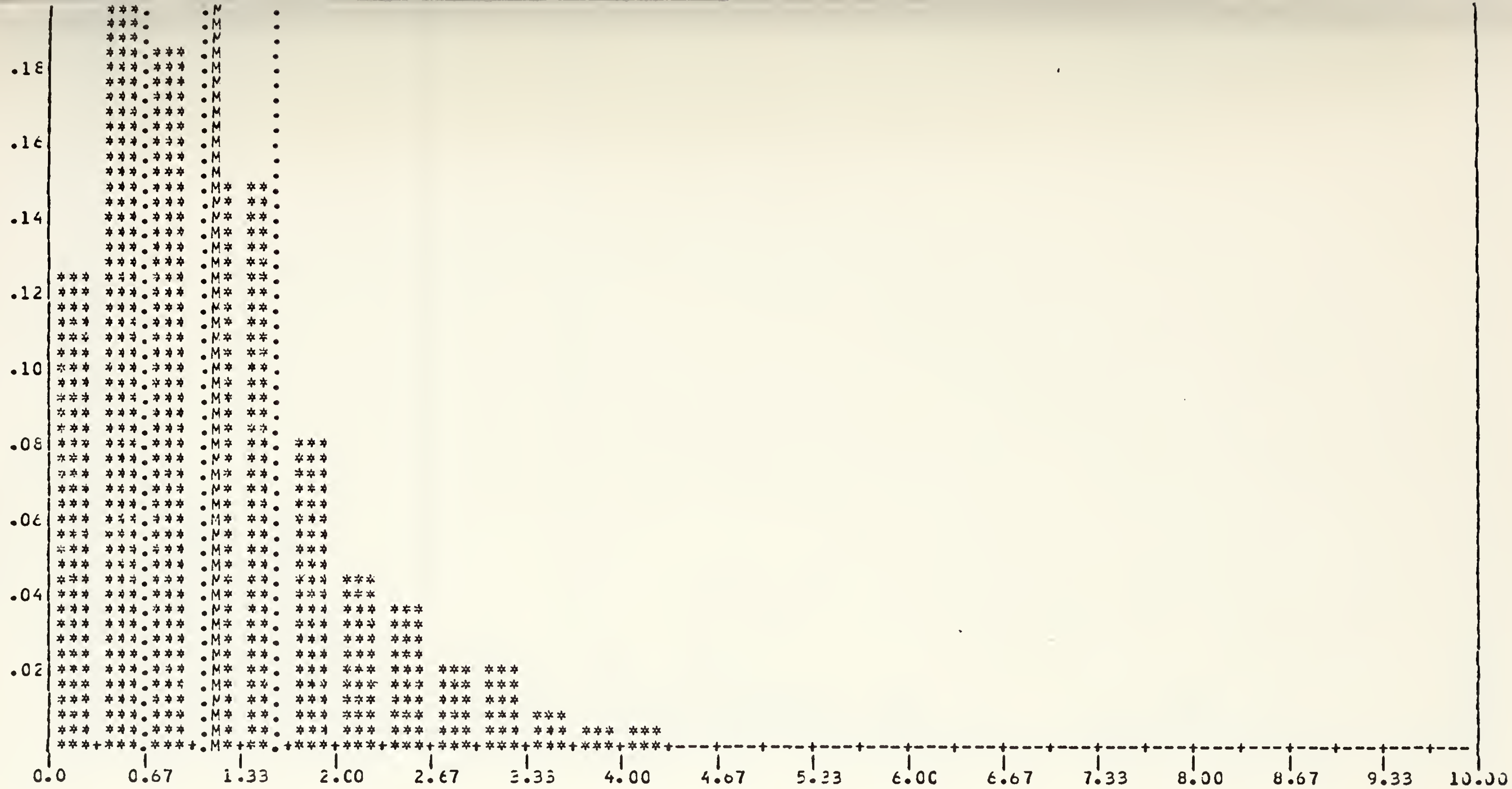
Substituting $\lambda = \frac{1}{m}(\ell n 2)^{1/\alpha}$

$$T = m \cdot \left(\frac{\ell n \mu}{\ell n 2}\right)^{\frac{1}{\alpha}} \sim \text{WEIBULL}\{\alpha, \frac{1}{m}(\ell n 2)^{1/\alpha}\}.$$

Following histograms in Figure III-1 show the effect of changing α in WEIBULL. As α decreases the distribution is widely spread and it is said that the distribution has heavy tail (Figure III-1, B). In the opposite case; i.e. α increases, it has light tail (Figure III-1, A).

Figure III-2 is observed keeping α fixed at 1 and median is 0.5, 1.0, 2.0 in WEIBULL.

This is exponential distribution with median 0.5, 1.0, 2.0.

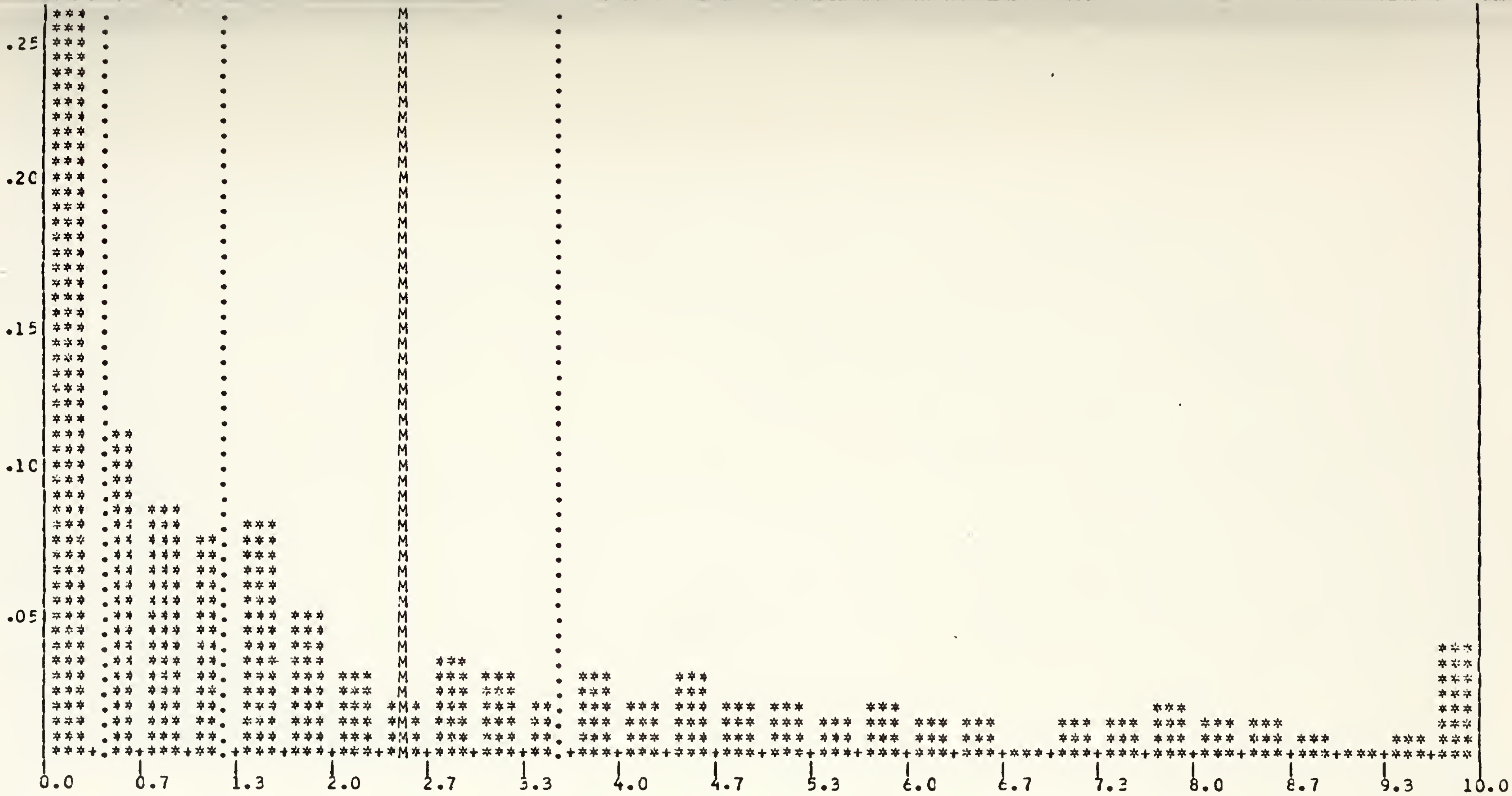


```
SCALE FIXED FROM  0.0          TO  1.000000E 01


CENTRAL TENDENCY          SPREAD                       HIGHER CENTRAL MOMENTS     DISTRIBUTION

MEAN        1.135735E 00  VARIANCE    5.680716E-01     M3          4.249120E-01   MINIMUM                        1.352569E-02
MEDIAN      9.871483E-01  STD DEV     7.537053E-01     M4          1.25[illegible]379E 00   .10 QUANTILE                   2.862445E-01
TRIMEAN     1.017548E 00  COEF VAR    6.636280E-01     SKEWNESS    9.924175E-01   .25 QUANTILE  (HINGE)          5.875416E-01
MIDMEAN     1.019394E 00  MEAN DEV    5.847039E-01     KURTOSIS    8.845930E-01   .50 QUANTILE  (MEDIAN)         9.871483E-01
MIDRANGE    2.078744E 00  RANGE       4.130438E 00     BETA1       4.223661E-01   .75 QUANTILE  (HINGE)          1.508355E 00
GEOM MEAN   8.636203E-01  MIDSPREAD  .9.208136E-01     BETA2       1.247411E 00   .90 QUANTILE                   2.199251E 00
HARM MEAN   5.009067E-01                                                          MAXIMUM                        4.143964E 00
```

FIGURE III-1A, WEIBULL{1.5, 0.7832}

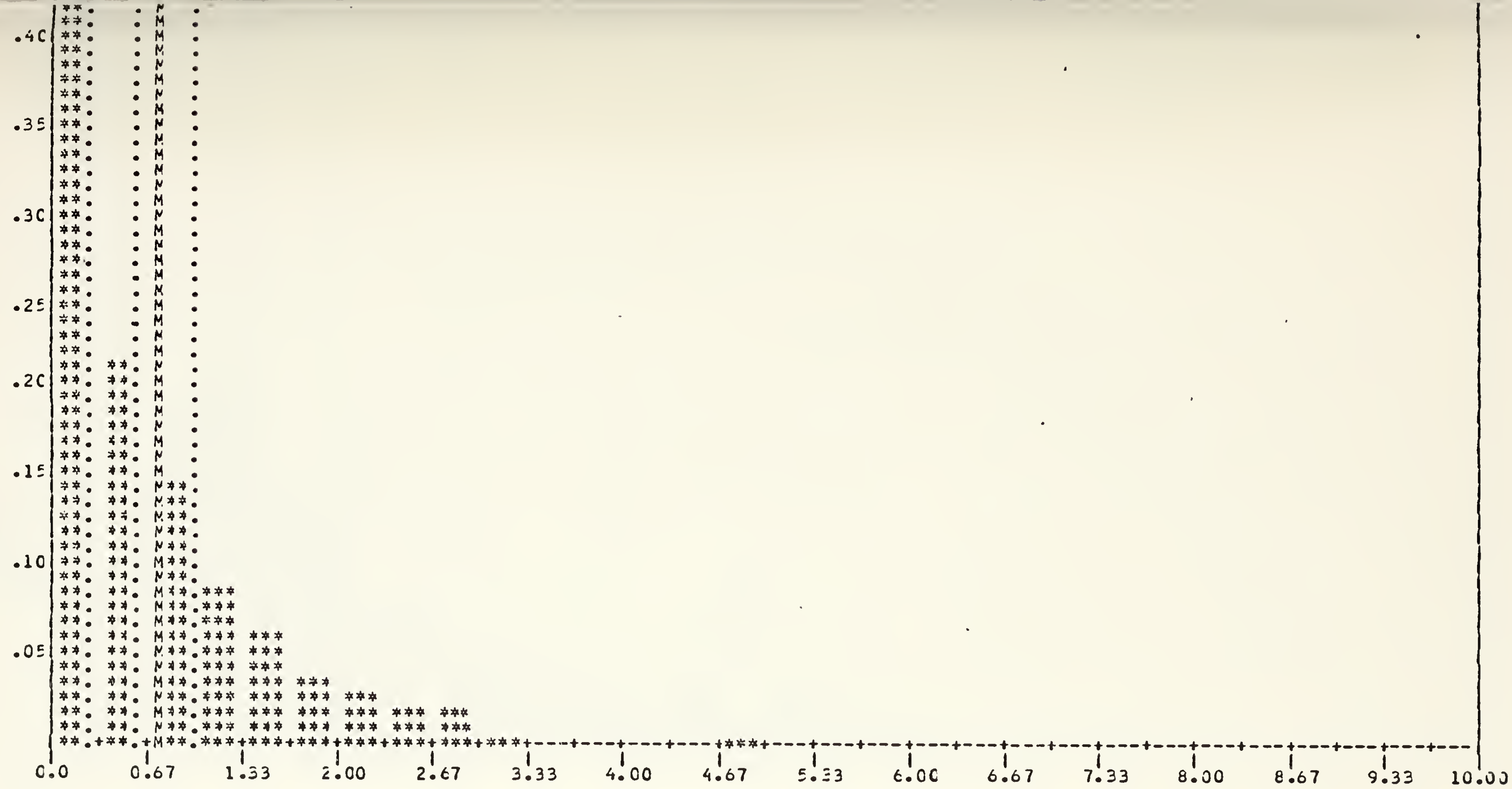


```
SCALE FIXED FROM  0.0          TO  1.000000E 01


CENTRAL TENDENCY            SPREAD                         HIGHER CENTRAL MOMENTS      DISTRIBUTION

MEAN        2.424064E 00    VARIANCE    1.078416E 01       M3          1.088363E 02    MINIMUM                          1.133222E-04
MEDIAN      1.174018E 00    STD DEV     3.283925E 00       M4          2.156061E 03    .10 QUANTILE                     9.521091E-02
TRIMEAN     1.551265E 00    COEF VAR    1.354718E 00       SKEWNESS    3.073224E 00    .25 QUANTILE  (HINGE)            3.201752E-01
MIDMEAN     1.361762E 00    MEAN DEV    2.010052E 00       KURTOSIS    1.553906E 01    .50 QUANTILE  (MEDIAN)           1.174018E 00
MIDRANGE    1.414110E 01    RANGE       2.828197E 01       BETA1       1.081842E 02    .75 QUANTILE  (HINGE)            3.536851E 00
GEOM MEAN   8.584681E-01    MIDSPREAD   3.216675E 00       BETA2       2.140204E 03    .90 QUANTILE                     6.528567E 00
HARM MEAN   2.588440E-02                                                               MAXIMUM                          2.828209E 01
```

FIGURE III-1B,{WEIBULL 0.7, 0.5923}

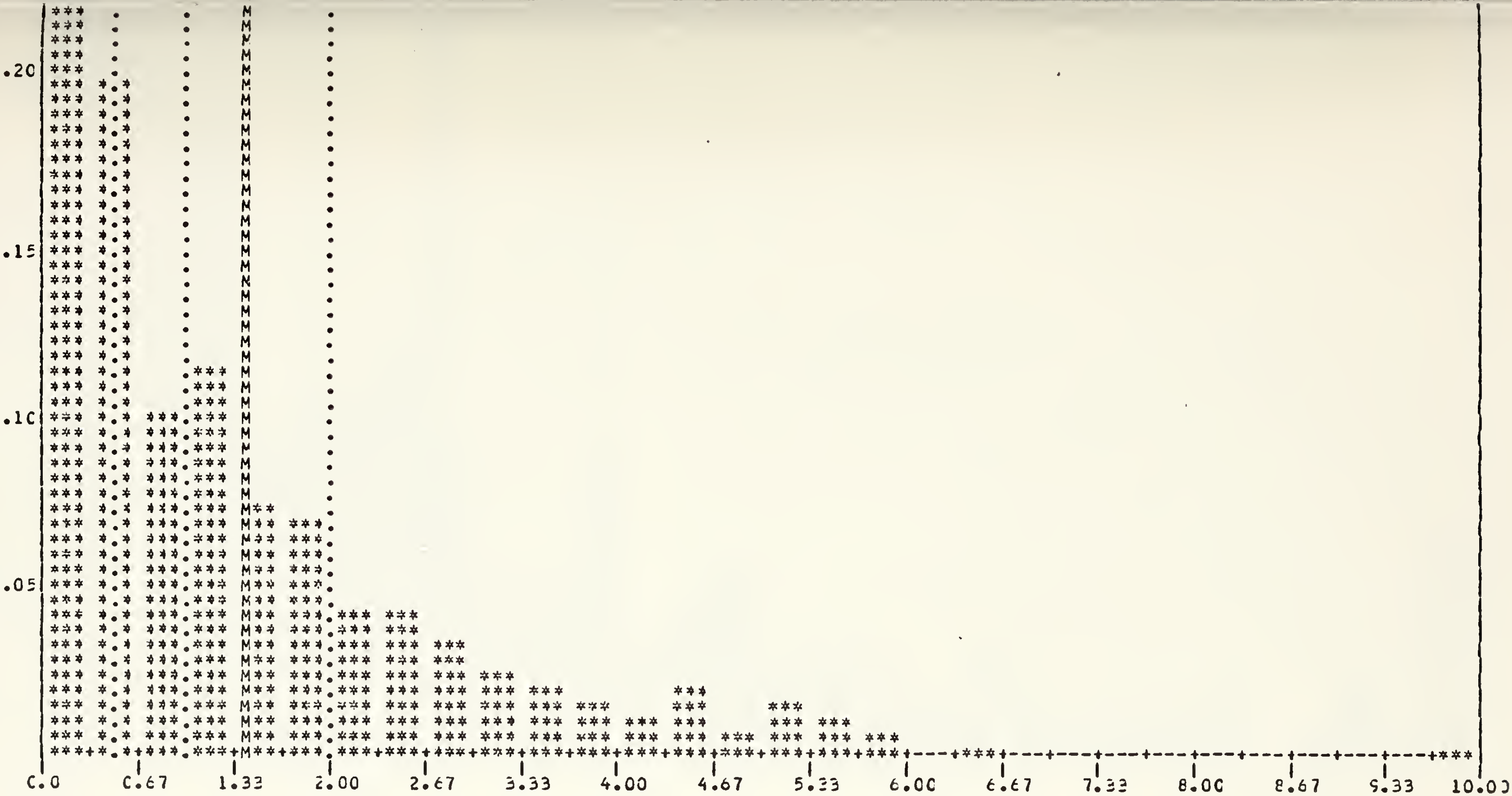


```
SCALE FIXED FROM  0.0          TO  1.000000E 01

CENTRAL TENDENCY              SPREAD                        HIGHER CENTRAL MOMENTS    DISTRIBUTION

MEAN        6.837079E-01      VARIANCE    4.609393E-01      M3         5.514978E-01   MINIMUM                    2.945260E-05
MEDIAN      4.750265E-01      STD DEV     6.789252E-01      M4         1.520003E 00   .10 QUANTILE               7.237494E-02
TRIMEAN     5.247900E-01      COEF VAR    9.930047E-01      SKEWNESS   1.762293E 00   .25 QUANTILE  (HINGE)      1.946158E-01
MIDMEAN     4.965945E-01      MEAN DEV    4.815852E-01      KURTOSIS   4.157899E 00   .50 QUANTILE  (MEDIAN)     4.750265E-01
MIDRANGE    2.481583E 00      RANGE       4.963107E 00      BETA1      5.481934E-01   .75 QUANTILE  (HINGE)      9.544921E-01
GEOM MEAN   3.847181E-01      MIDSPREAD   7.598763E-01      BETA2      1.511179E 00   .90 QUANTILE               1.606824E 00
HARM MEAN   1.347680E-02                                                              MAXIMUM                    4.963137E 00
```

FIGURE III-2A, WEIBULL{1,1.3863}

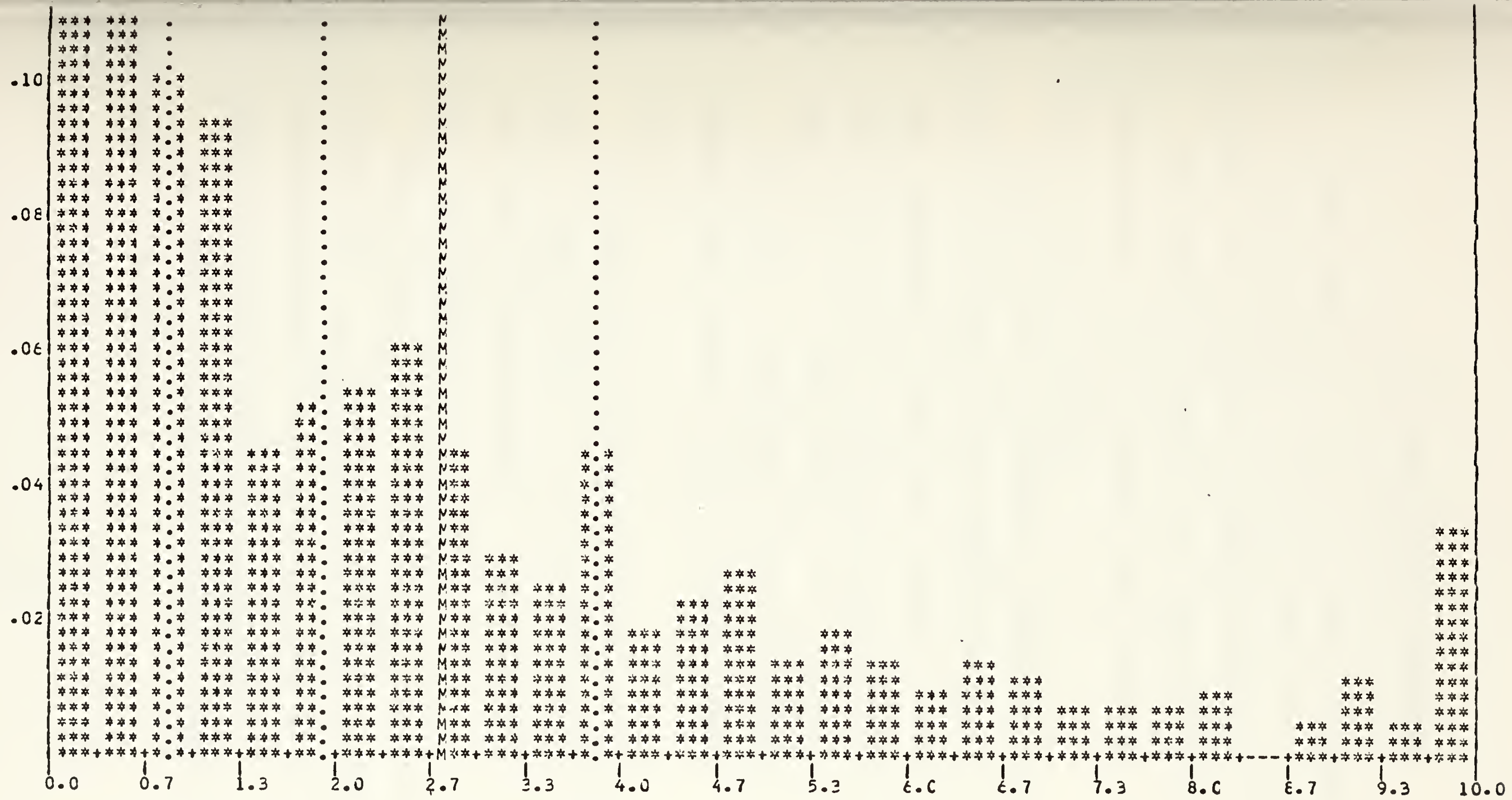


```
SCALE FIXED FROM  0.0          TO  1.000000E 01

CENTRAL TENDENCY           SPREAD                        HIGHER CENTRAL MOMENTS    DISTRIBUTION

MEAN        1.367415E 00   VARIANCE    1.843758E 00      M3         4.411988E 00   MINIMUM                       5.890521E-05
MEDIAN      9.500527E-01   STD DEV     1.357850E 00      M4         2.431387E 01   .10 QUANTILE                  1.447499E-01
TRIMEAN     1.049580E 00   COEF VAR    9.930047E-01      SKEWNESS   1.762296E 00   .25 QUANTILE (HINGE)          3.892317E-01
MIDMEAN     9.931891E-01   MEAN DEV    9.631665E-01      KURTOSIS   4.157902E 00   .50 QUANTILE (MEDIAN)         9.500527E-01
MIDRANGE    4.963165E 00   RANGE       9.926214E 00      BETA1      4.385553E 00   .75 QUANTILE (HINGE)          1.908984E 00
GEOM MEAN   7.694359E-01   MIDSPREAD   1.519753E 00      BETA2      2.417888E 01   .90 QUANTILE                  3.213649E 00
HARM MEAN   2.695360E-02                                                           MAXIMUM                       9.926273E 00
```

FIGURE III-2B, WEIBULL{1,0.6931}



```
SCALE FIXED FROM  0.0          TC  1.000000E 01


CENTRAL TENDENCY              SPREAD                        HIGHER CENTRAL MOMENTS        DISTRIBUTION

MEAN        2.734832E 00      VARIANCE    7.375031E 00      M3          3.529585E 01      MINIMUM                     1.178104E-04
MEDIAN      1.900105E 00      STD DEV     2.715701E 00      M4          3.893254E 02      .10 QUANTILE                2.894998E-01
TRIMEAN     2.099160E 00      COEF VAR    9.930047E-01      SKEWNESS    1.762291E 00      .25 QUANTILE (HINGE)        7.784634E-01
MIDMEAN     1.986378E 00      MEAN DEV    1.926332E 00      KURTOSIS    4.157890E 00      .50 QUANTILE (MEDIAN)       1.900105E 00
MIDRANGE    9.926323E 00      RANGE       1.985242E 01      BETA1       3.508437E 01      .75 QUANTILE (HINGE)        3.817968E 00
GEOM MEAN   1.538872E 00      MIDSPREAD   3.039505E 00      BETA2       3.868616E 02      .90 QUANTILE                6.427298E 00
HARM MEAN   5.390721E-02                                                                  MAXIMUM                     1.985254E 01
```

FIGURE III-2C, WEIBULL{1,0.3466}

IV. RESULTS AND COMMENTS

Simulation of the four tests were performed 5,000 times (5,000 replications). In each replication H_o: $CEP^2=1$; H_1: $CEP^2=2$; and Type I, II error rates are 0.05.

The following tables IV-1,2,3 show the mean number of terminations, their variances, and number of acceptances of H_o in 5,000 replications.

Table IV-1 shows the effects of changing median values, where the shape parameter of the WEIBULL variate was fixed at 1.0 (the exponential variate with changing median). Table IV-2 and 3 shows the effects of changing shape parameter, the median being fixed at 1.0 in table IV-2 and at 2.0 in table IV-3.

For ease of comparison these were graphed. Each figure contains four curves and each curve represents the case A, B, C or D.

In testing the difference of E[N], null hypothesis would be $E[N_1] = E[N_2]$, where N_1, N_2 are random variables representing the number of termination by test cases which one wishes to test the difference of the E[N].

By the central limit theorem [2] the difference in E[N] greater than $\frac{1.96 \times \hat{\sigma}}{n}$ is significant at level 0.05. $\hat{\sigma}$ is estimated standard deviation of N. But it is different between the test cases being considered.

The larger one might be selected so that we may not reject H_o erroneously.

THIS CASE IS OBSERVED CHANGING MEDIAN AND SHAPE PARAMETER IS FIXED AT 1.

MEDIAN	CASE A			CASE B			CASE C			CASE C		
	E(N)	V(N)	# ACCEPT	E(N)	V(N)	# ACCEPT	E(N)	V(N)	# ACCEPT	E(N)	V(N)	# ACCEPT
0.54	7.035	2.930	5000.	10.382	17.428	5000.	10.294	6.230	4999.	11.874	4.796	5000.
0.67	8.350	7.249	4996.	13.605	47.889	4997.	11.830	15.537	4998.	13.151	14.061	4997.
0.84	10.646	23.490	4950.	19.569	156.983	4953.	15.145	58.313	4941.	16.064	49.647	4936.
1.00	13.728	63.682	4745.	28.178	408.287	4771.	19.803	142.636	4752.	20.176	131.639	4744.
1.08	15.470	94.006	4487.	33.284	620.852	4484.	22.603	235.429	4489.	22.914	214.990	4538.
1.19	17.843	174.842	3944.	39.552	967.506	3939.	25.780	344.298	3957.	26.317	349.988	4030.
1.27	18.444	189.203	3426.	43.717	1232.651	3404.	27.081	419.085	3474.	27.707	438.859	3508.
1.33	18.805	201.792	2898.	45.522	1328.232	2917.	28.445	504.042	2987.	28.615	484.615	3027.
1.39	18.892	220.636	2530.	46.051	1360.952	2468.	28.027	507.703	2545.	28.248	487.113	2575.
1.40	18.395	214.427	2418.	47.216	1362.500	2433.	27.763	473.886	2399.	27.808	493.739	2506.
1.40	18.199	208.286	2284.	46.197	1358.806	2369.	28.192	510.770	2381.	27.746	496.408	2448.
1.47	17.468	195.246	1971.	45.525	1279.995	1938.	27.178	487.229	1945.	27.320	496.467	2026.
1.55	16.244	186.430	1423.	43.445	1168.529	1423.	25.548	440.333	1485.	25.638	447.553	1524.
1.65	14.138	133.647	951.	39.535	908.235	909.	22.901	349.643	966.	22.891	351.993	955.
1.82	11.882	96.718	501.	34.765	603.266	475.	19.117	215.990	470.	18.667	226.886	503.
2.00	9.432	63.624	257.	30.126	428.890	236.	15.978	153.003	234.	15.529	157.854	226.
2.45	6.453	26.128	58.	21.725	156.938	70.	11.052	54.749	70.	10.482	56.675	42.
2.67	5.476	17.995	37.	20.003	124.732	33.	9.810	39.096	32.	9.135	41.098	23.
3.25	4.128	9.233	6.	16.338	62.552	7.	7.792	20.598	5.	7.055	17.663	4.
3.45	3.843	7.617	3.	15.635	53.751	4.	7.282	15.668	1.	6.647	14.994	5.

TABLE IV-1

```
THIS CASE IS OBSERVED CHANGING SHAPE PARAMETER AND MEDIAN IS FIXED AT 1.

MEDIAN         CASE A                      CASE B                        CASE C                      CASE D
          E(N)     V(N)   # ACCEPT    E(N)     V(N)   # ACCEPT     E(N)     V(N)   # ACCEPT    E(N)     V(N)   # ACCEPT

0.50     6.139   23.478   1001.     27.919  399.861   4742.      23.350  374.262   1057.     18.571  227.114    492.

0.60     8.654   46.990   1569.     27.818  418.116   4738.      27.007  442.982   1828.     24.335  382.756   1161.

0.70    11.147   73.146   2389.     27.631  398.115   4715.      26.510  525.863   2866.     27.450  485.972   2274.

0.80    13.616   99.977   3405.     27.516  381.577   4722.      26.742  390.984   3794.     28.004  474.136   3488.

0.90    14.419   90.972   4277.     28.033  400.513   4726.      22.999  250.668   4420.     24.362  268.340   4341.

1.00    13.728   63.682   4745.     28.178  408.287   4771.      19.803  142.636   4752.     20.176  131.639   4744.

1.10    12.799   40.668   4948.     28.351  435.550   4766.      16.602   81.464   4891.     17.214   70.285   4898.

1.20    11.679   24.907   4982.     28.020  396.977   4731.      14.586   47.508   4959.     15.006   34.312   4977.

1.30    10.907   15.607   5000.     28.154  421.004   4771.      13.094   26.650   4982.     13.421   16.557   4992.

1.40    10.420   11.415   5000.     28.234  412.444   4746.      11.892   17.057   4998.     12.621   10.211   4999.

1.50    10.056    8.504   5000.     27.924  391.072   4712.      11.043   10.840   4998.     11.950    5.687   5000.
```

TABLE IV-2

THIS CASE IS OBSERVED CHANGING SHAPE PARAMETER AND MEDIAN IS FIXED AT 2.

MEDIAN	CASE A			CASE B			CASE C			CASE D		
	E(N)	V(N)	# ACCEPT	E(N)	V(N)	# ACCEPT	E(N)	V(N)	# ACCEPT	E(N)	V(N)	# ACCEPT
0.50	3.973	10.394	206.	46.124	1307.159	2250.	12.237	77.664	86.	9.791	44.278	30.
0.60	4.690	15.043	193.	44.425	1221.383	1603.	13.193	93.457	94.	10.442	55.562	41.
0.70	5.585	21.765	230.	41.048	952.357	1096.	13.433	97.599	132.	11.550	71.607	66.
0.80	6.741	32.653	225.	37.358	766.110	685.	14.326	110.328	150.	12.367	88.580	103.
0.90	8.083	45.046	220.	33.644	558.666	376.	15.105	130.406	181.	13.781	108.805	178.
1.00	9.432	63.624	257.	30.126	428.890	236.	15.978	153.003	234.	15.529	157.854	226.
1.10	11.371	83.627	321.	26.460	297.943	171.	17.194	176.091	267.	17.012	188.225	355.
1.20	13.104	112.699	302.	24.219	210.740	101.	17.999	169.367	348.	19.005	225.854	504.
1.30	15.630	159.630	304.	21.774	161.597	58.	19.049	222.104	407.	20.867	285.435	731.
1.40	18.170	222.499	352.	20.191	122.917	28.	20.203	256.703	620.	23.113	357.848	1035.
1.50	21.008	290.951	394.	18.723	100.249	22.	21.274	282.634	691.	25.514	415.375	1367.

TABLE IV-3

REFERENCE FOR THE FIGURES IV-1 - IV-9

————————	CASE A
+ ——— + ——— +	CASE B
* ——— * ——— *	CASE C
x ——— x ——— x	CASE D

FIGURE IV-1, 4, 7;	Expected Sample Sizes
FIGURE IV-2, 5, 8;	Variances of the Sample Sizes
FIGURE IV-3, 6, 9;	Operating Characteristic Curves

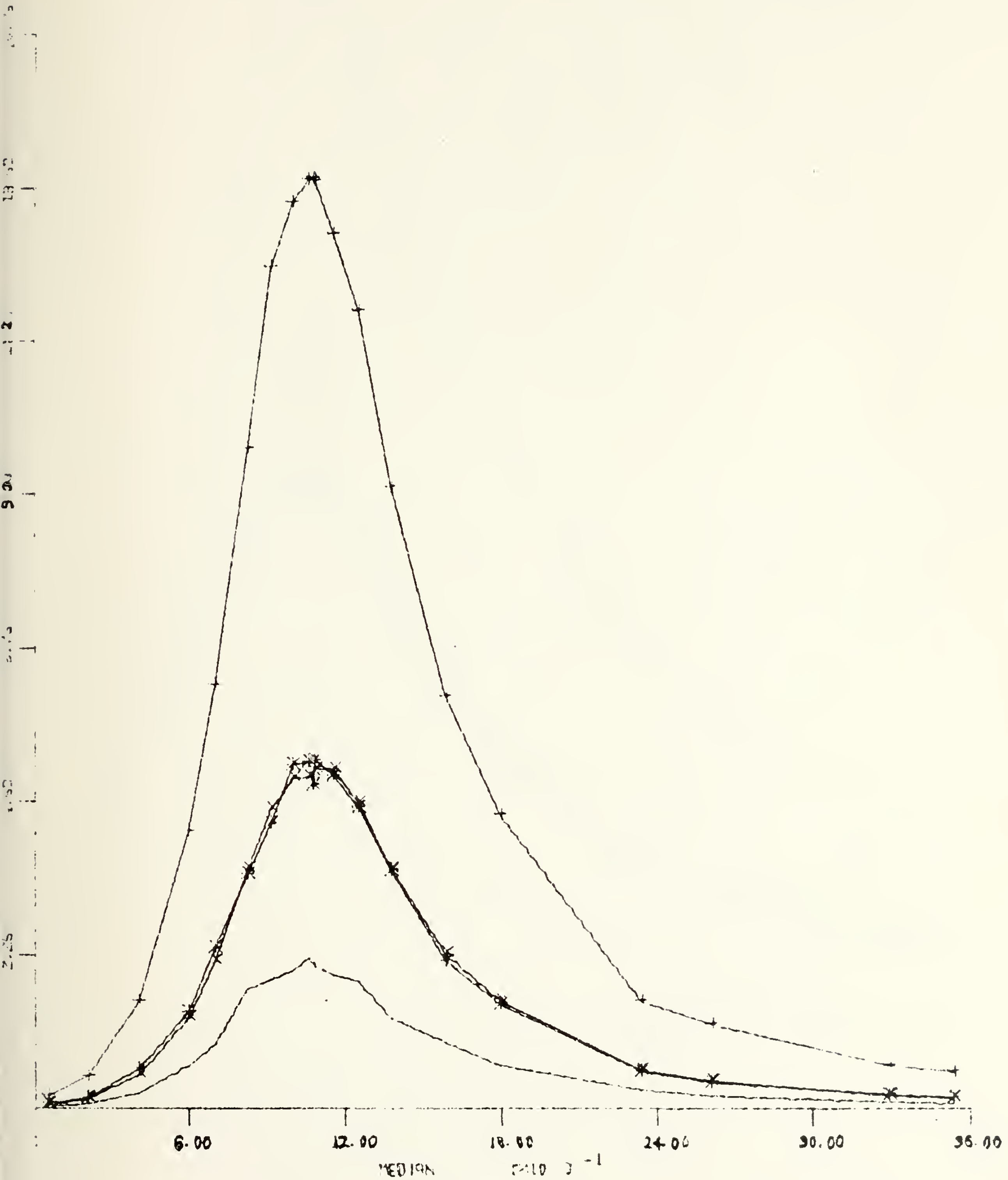


FIGURE IV-1

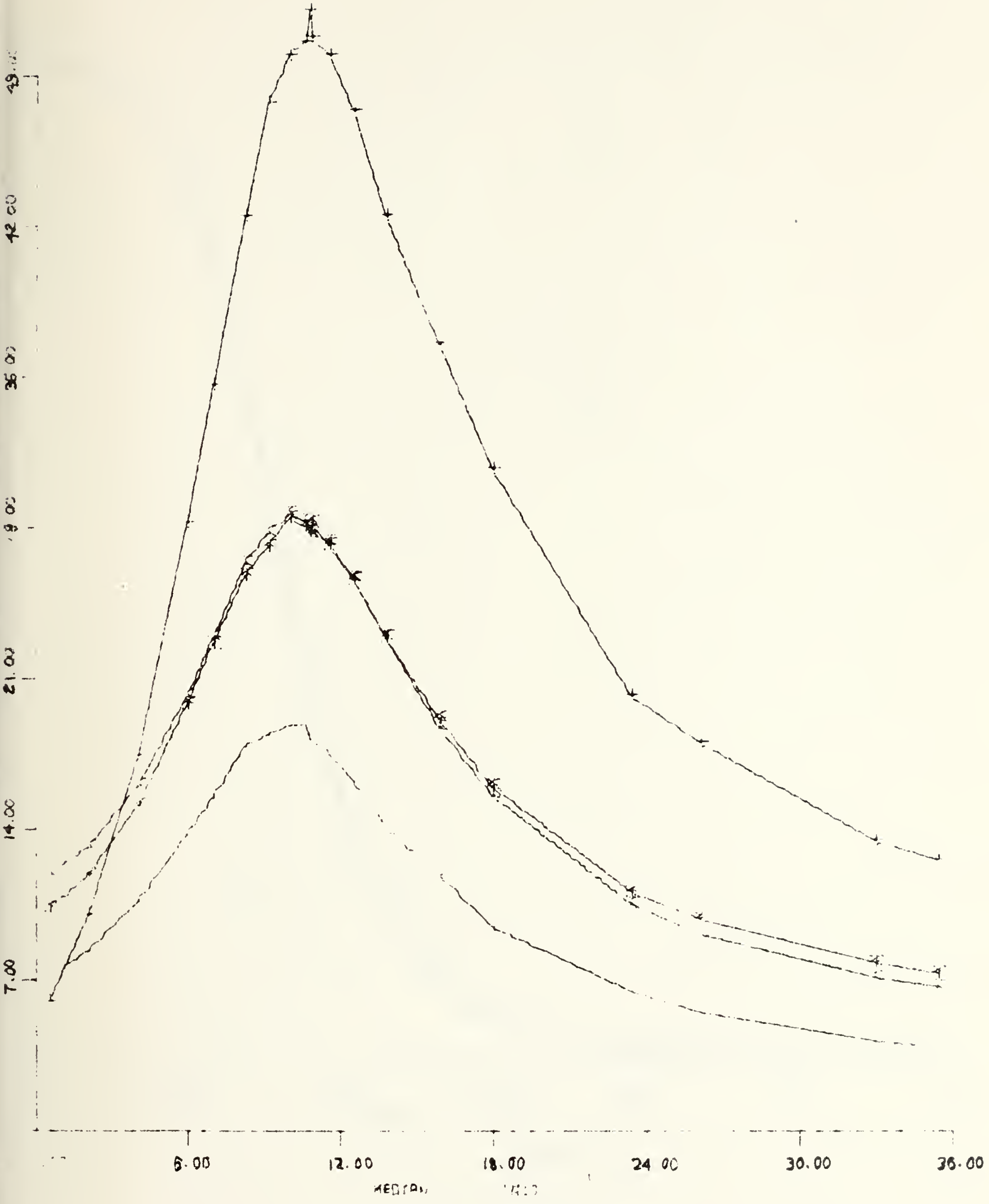

7.00
14.00
21.00
28.00
35.00
42.00
49.00
6.00
12.00
18.00
24.00
30.00
36.00
MEDIAN

FIGURE IV-2

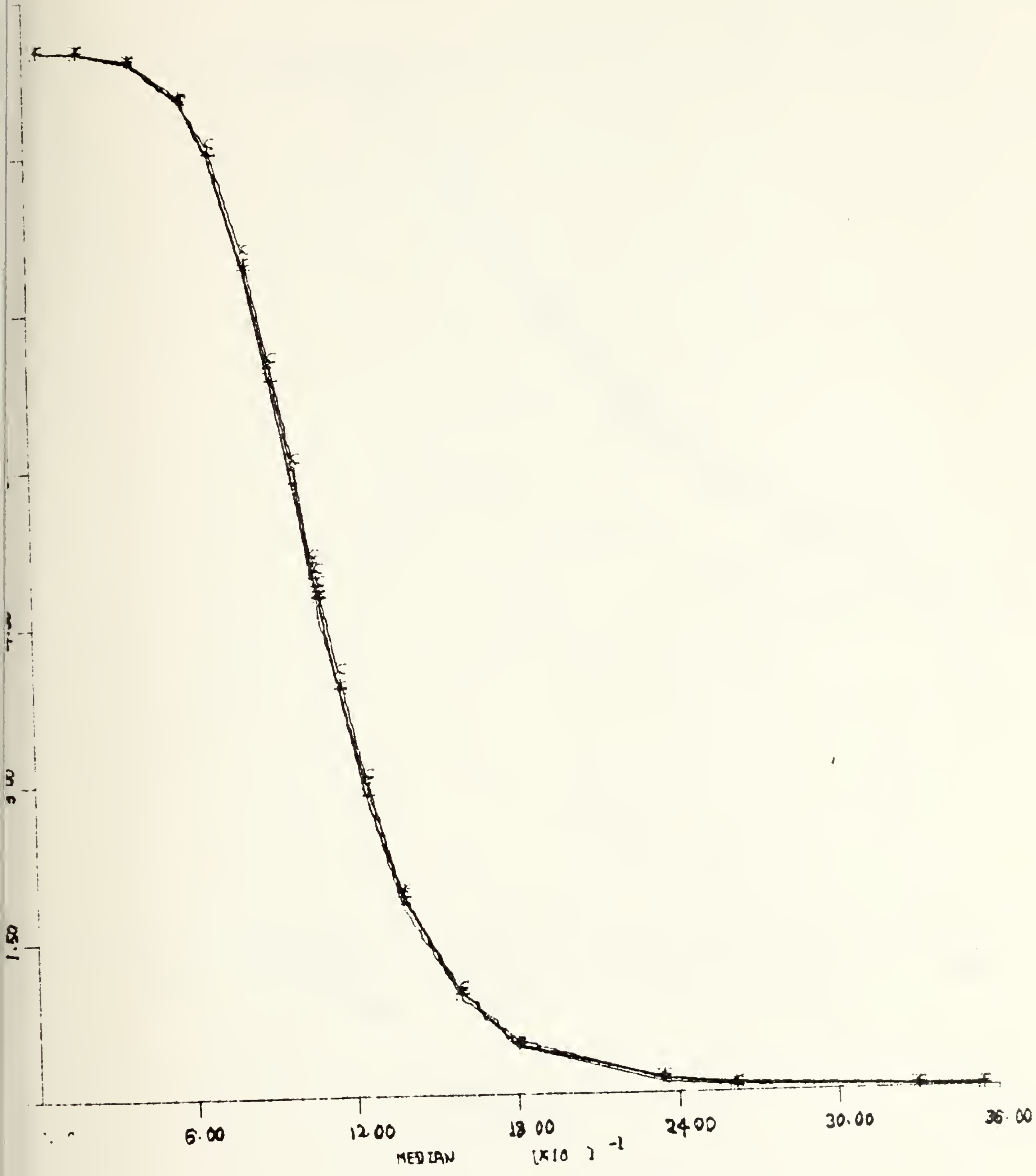


FIGURE IV-3

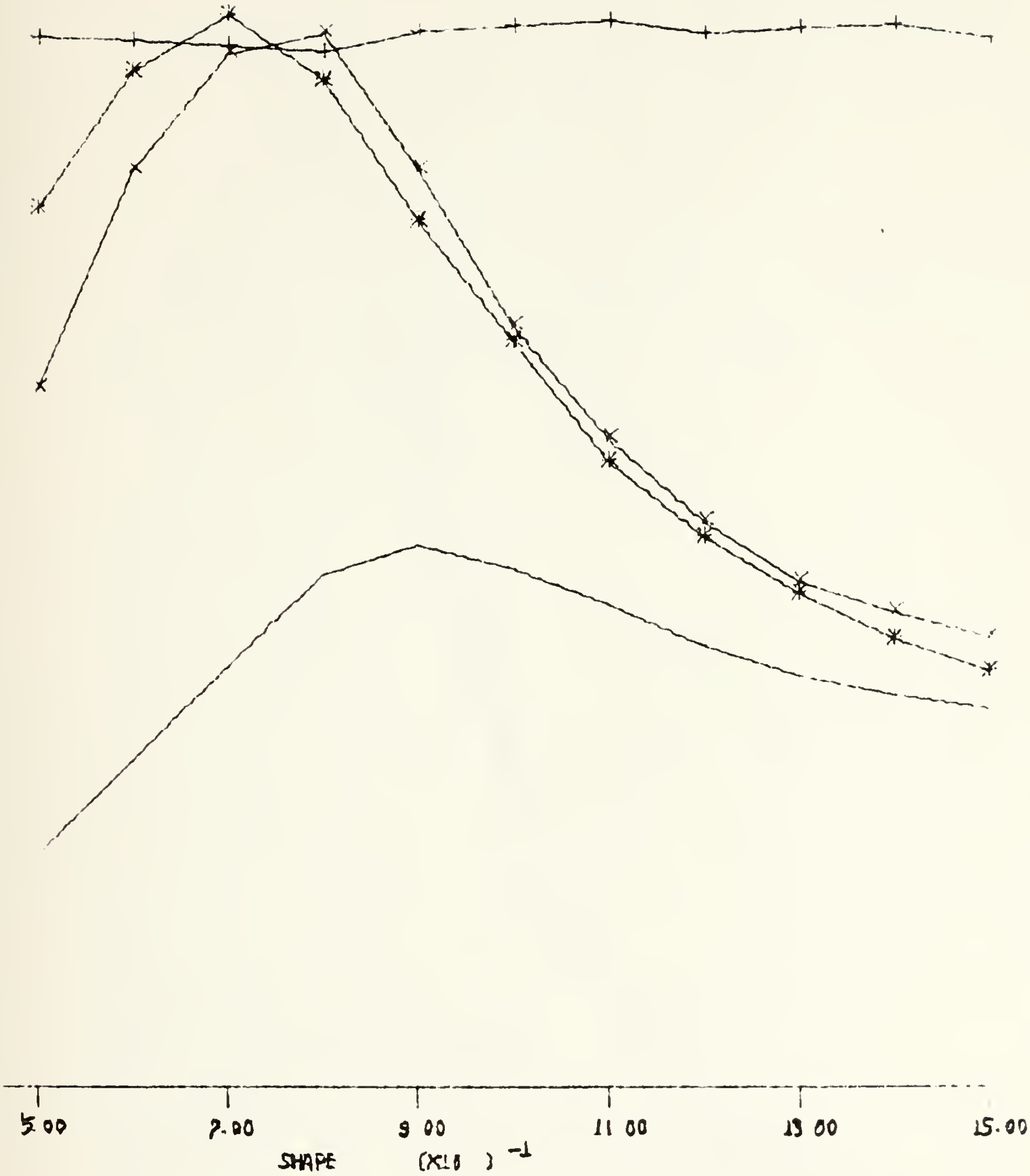


FIGURE IV-4

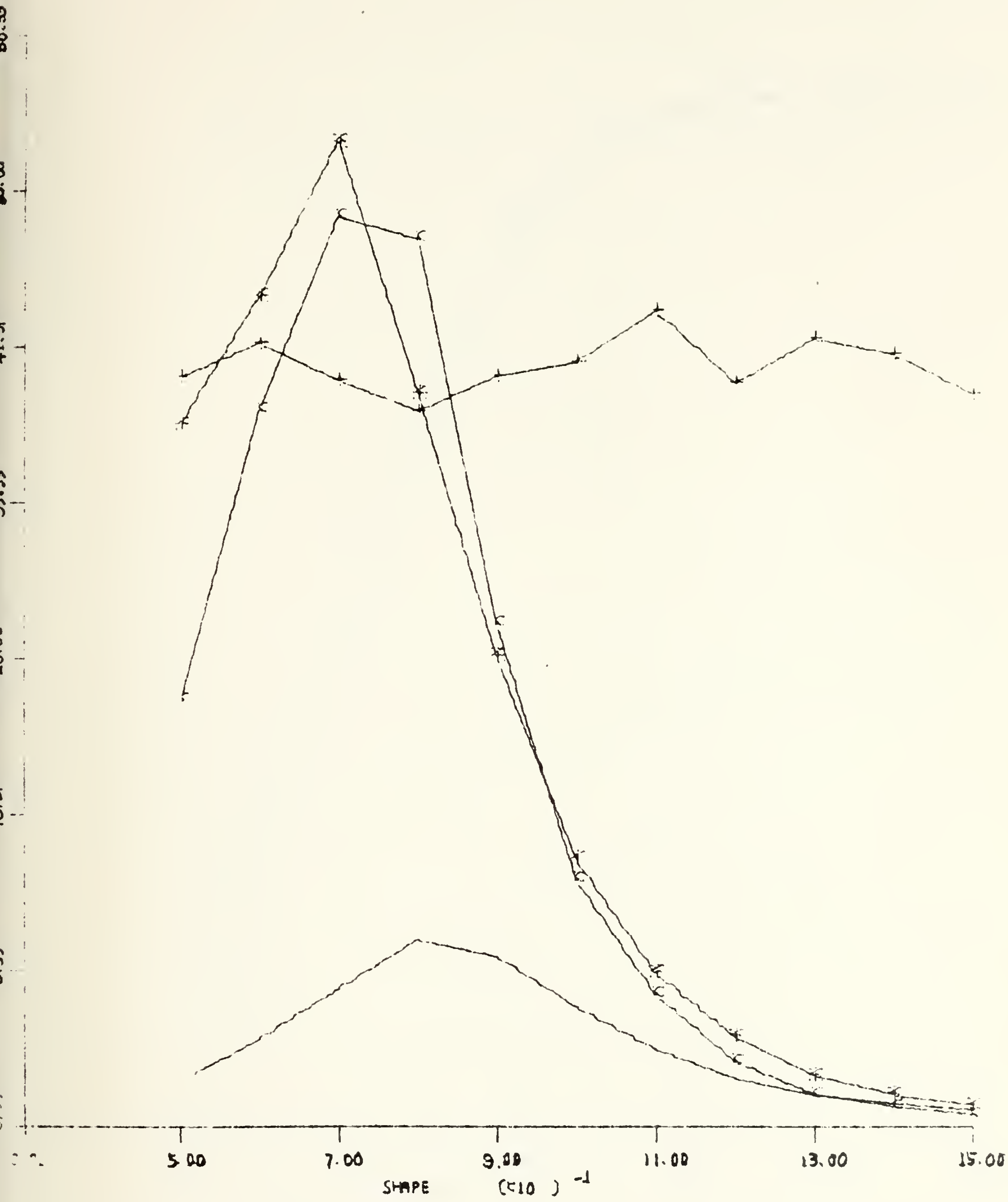

58.33
50.00
41.67
33.33
25.00
16.67
8.33
0.00
3.00
5.00
7.00
9.00
11.00
13.00
15.00
SHAPE (*10) -1

FIGURE IV-5

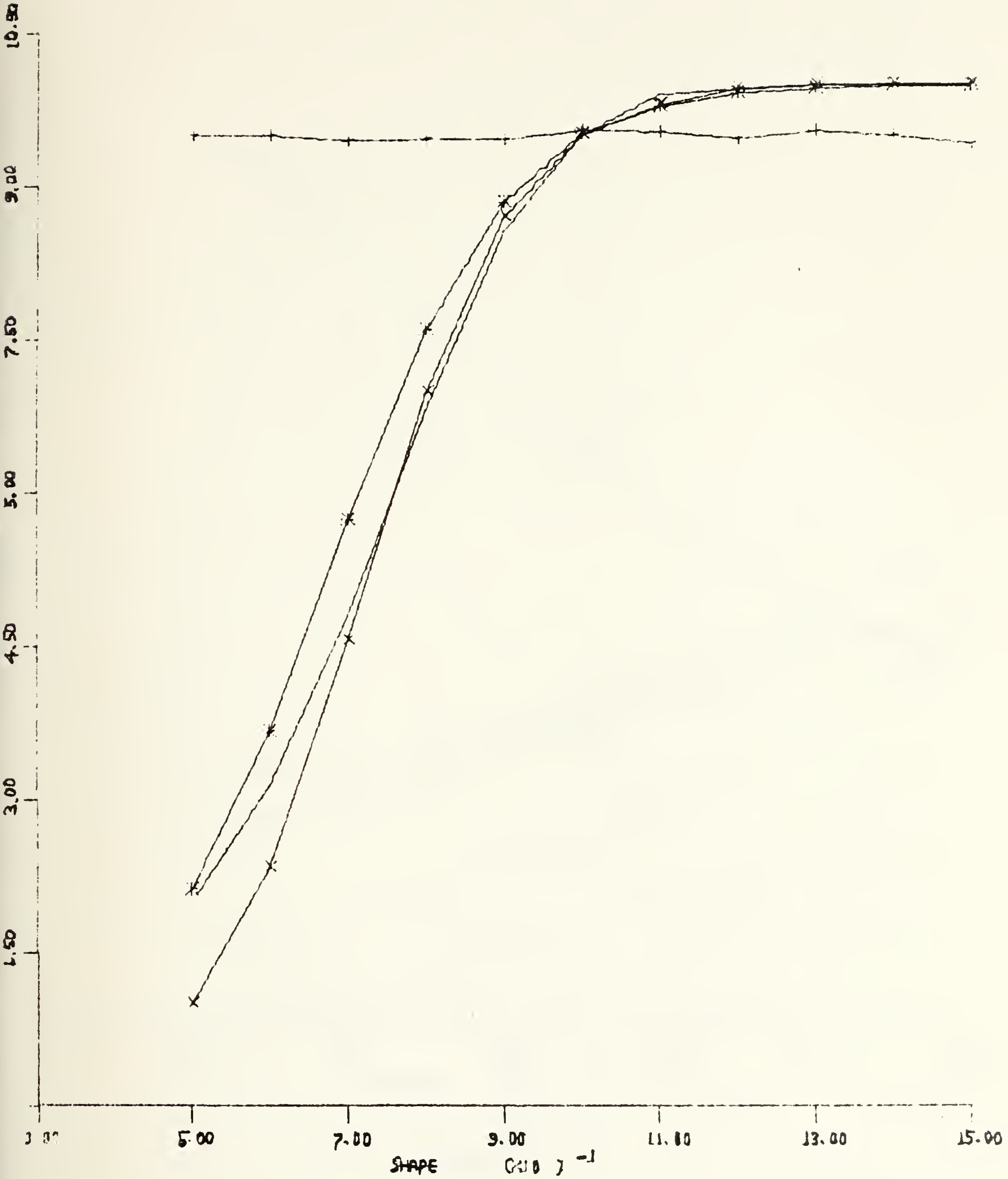

10.50
9.00
7.50
6.00
4.50
3.00
1.50
L(C) (X10)
3.00
5.00
7.00
9.00
11.00
13.00
15.00
SHAPE (X10) -1

FIGURE IV-6

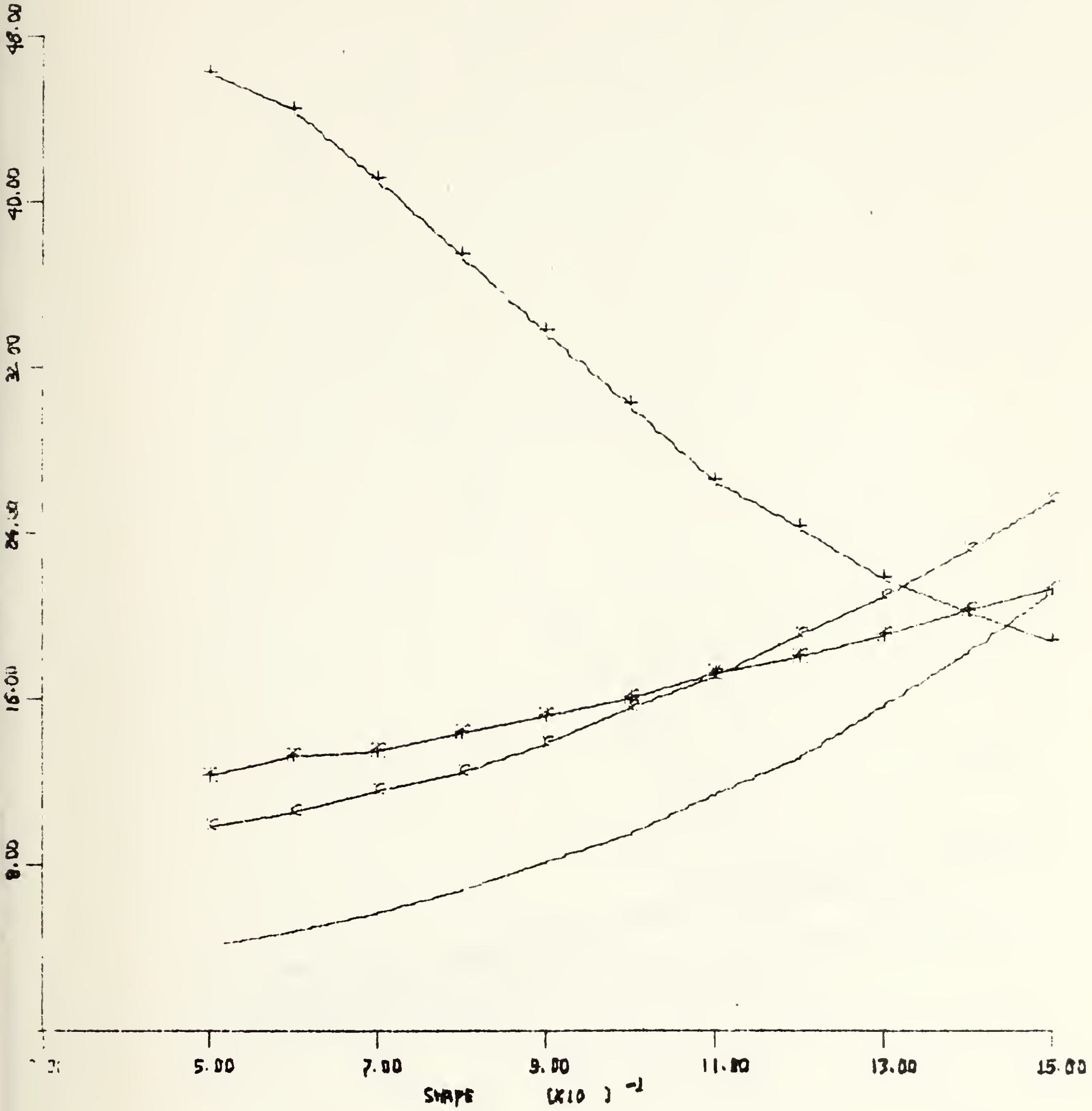

48.00
40.00
32.00
24.00
16.00
8.00
5.00
7.00
9.00
11.00
13.00
15.00
SHAPE (X10) -1

FIGURE IV-7

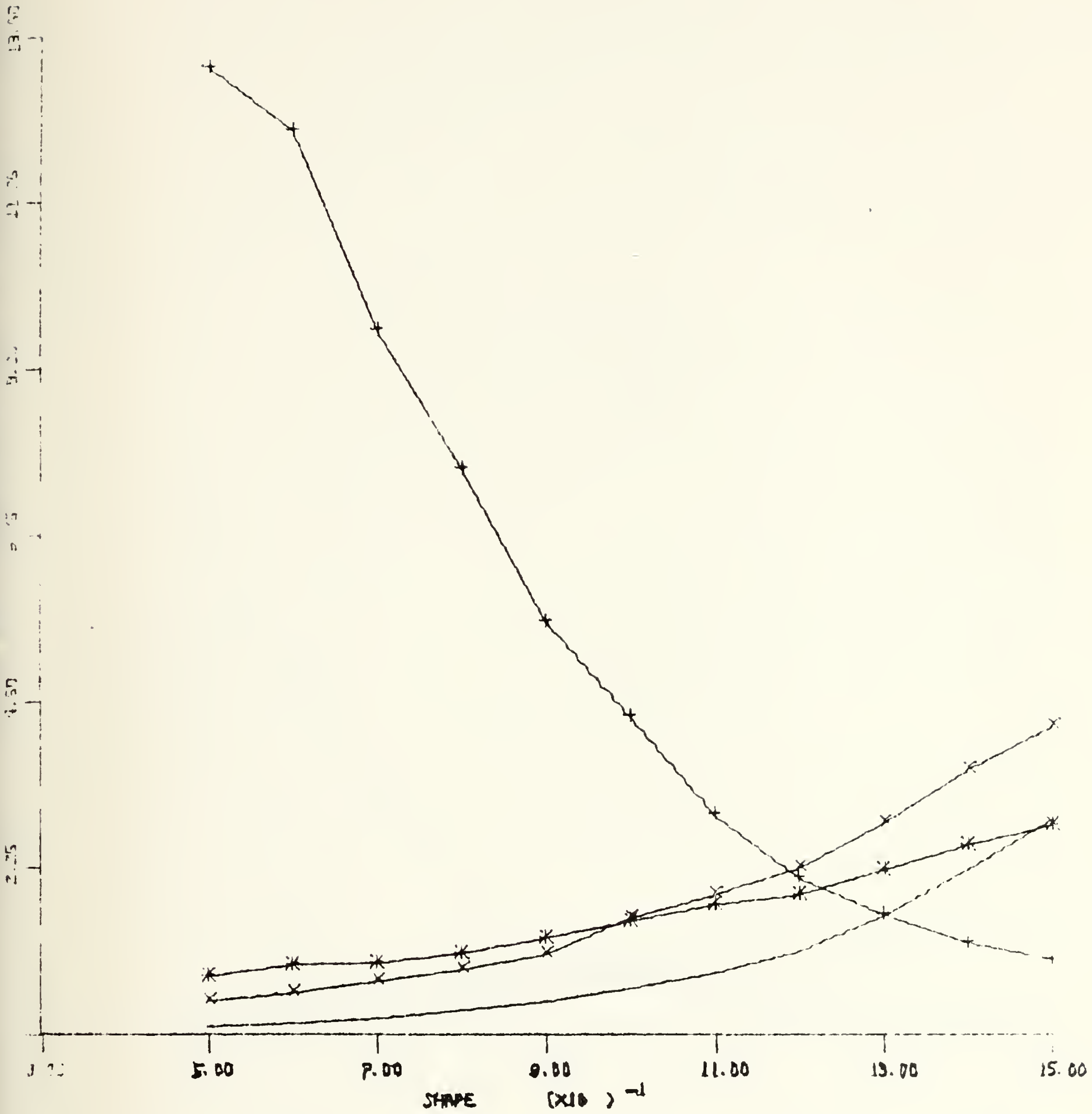

13.50
11.25
9.00
6.75
4.50
2.25
VAR(N) (X10)
3.00
5.00
7.00
9.00
11.00
13.00
15.00
SHAPE (X10) -1

FIGURE IV-8

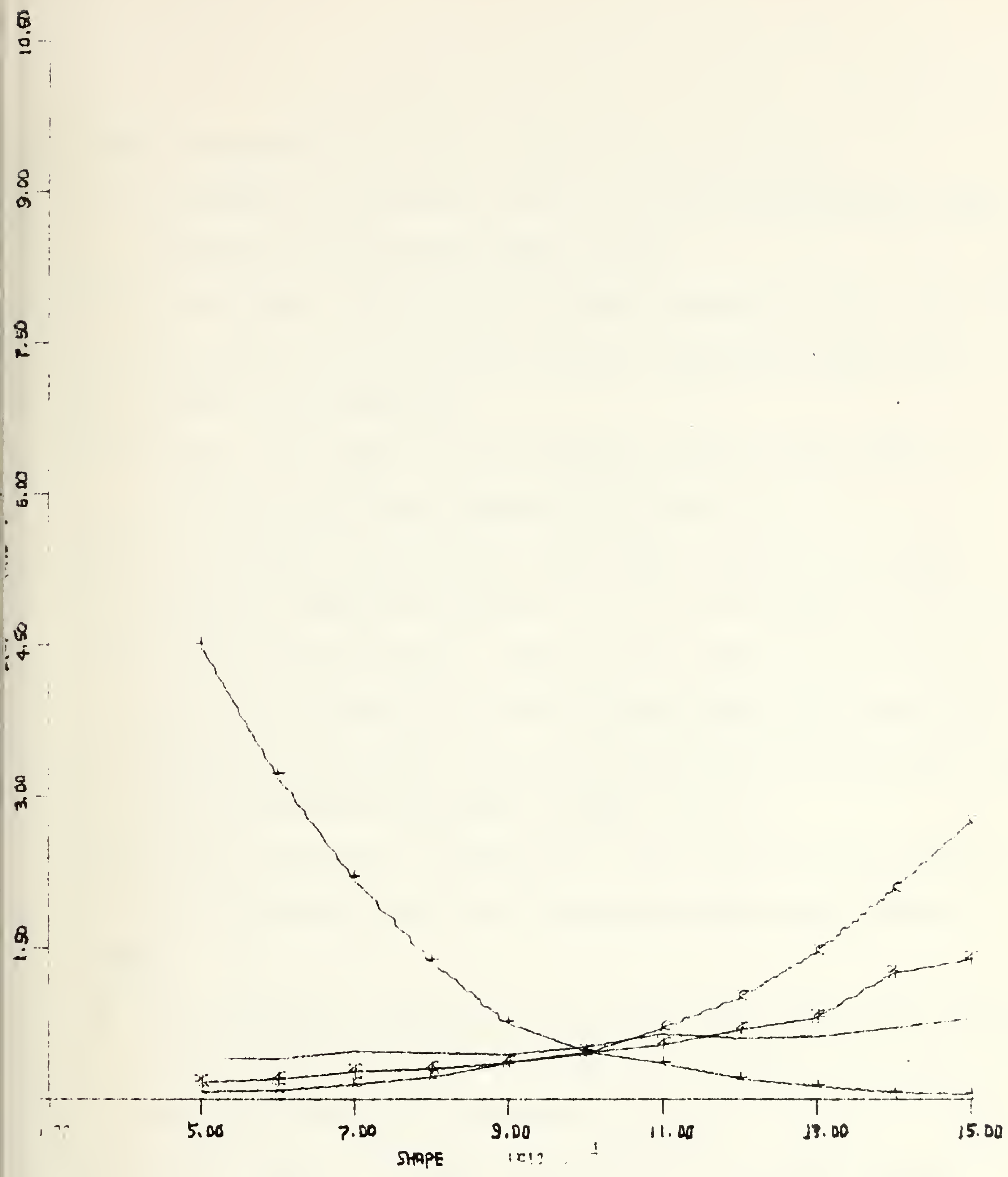


FIGURE IV-9

The interpretation of the figures is as follows:

a. Variances or standard deviations are approximately proportional to expected sample sizes.

b. Test Case A contains the minimum expected sample size and Test Case B includes the maximum expected sample size in almost all instances.

c. Case C and Case D yield similar results when median is changing and shape parameter is fixed at 1.0.

d. Test Case B is invariant about the change of skewness when the underlying median is 1.0 (See Figures IV-4,5,6).

e. Case B also gives the lowest error rates if the bomb impact has a heavy tail distribution under the alternative hypothesized median, and has light tail distribution under null hypothesized median (Figure IV-6.9).

f. Case C gives the lowest error rate when the true median is 2.0 and bomb impacts are clustered around the median.

NOTES:

a. The adjusting factors of the error rates table were developed iteratively and there is no guarantee that these adjusting factors are the best ones.

b. There could be further investigation of effects under error rates other than 0.05, or of changing null and alternative hypothesized median values.

c. The Fortran program for this simulation is attached as an Appendix.

V. CONCLUSION

If it is certain that the bomb impact is Rayleigh distributed, then Sequential Rayleigh Test is appropriate to test the system.

In case one has doubt about the bomb impact distribution, the Sequential Binomial Test gives better tests than the Sequential Rayleigh Test.

Further, if there is some reason to believe that the median is likely to be 1 (so the test is likely to accept H_o), Case B (Null parameter $P_o=0.5$) is better than any of the other three tests.

If the median is not likely to be 1, test case A (Sequential Rayleigh Test) or test case C (which has null parameter that minimized E[N] under H_o) are the best tests.

APPENDIX

```
      DIMENSION BOUND(300,2,4),DATA(4,4),TEST(5000,2,4)
C      BOUND----CONTAINS DECISION BOUNDARIES FOR EACH STAGES
C     TEST----FIRST ELEMENT REPRESENTS NUMBER OF REPLICATION
C             SECOND ELEMENT REPRESENTS DECISION
C             THIRD ELEMENT REPRESENTS TEST CASE(A,B,C OR D)
C     DATA----TEST RESULT,FIRST ROW---MEAN NUMBER OF TERMINA
C        TION SECOND ROW---VARIANCE,THIRD ROW---NUMBER ACCEP
C       TANCE FOURTH ROW---NUMBER REJECTION
        ISEED= 688777
        NCOUNT=5000
      READ(5,2)ALPHA,BETA,C0,C1
C     C0------MEDIAN UNDER THE NULL HYPCTHESIS
C     C1------MEDIAN UNDER THE ALTERNATIVE HYPOTHESIS
       WRITE(6,140)ALPHA,BETA,C0,C1
  140  FORMAT(5X,'ALPHA=',F6.3,'    BETA=',F6.3,'    C0=',F6.3
     *,'    C1=',F6.3)
       R=-C0*ALOG(.1584)/(ALOG(2.))
C     R----HIT MISS CRITERION IN CASE C
C     R2----HIT MISS CRITERION IN CASE D
       P40=.898657
       R2=(ALOG(1-P40))/ALOG(.5)
       P41=1.-1./2**(R2/2.)
    2 FORMAT(4F6.3)
C COMPUTE BOUND OF REJECTICN AND ACCEPTANCE
       B1=ALOG(1.2*BETA/(1-2.*ALPHA))
       A1=ALOG((1-1.2*BETA)/(2.*ALPHA))
       B2=ALOG(1.2*BETA/(1-1.2*ALPHA))
       A2=ALOG((1.-1.2*BETA)/(1.2*ALPHA))
       B3=ALOG(1.15*BETA/(1-1.5*ALPHA))
       A3=ALOG((1-1.15*BETA)/(1.5*ALPHA))
       B4=ALOG(1.1*BETA/(1-1.5*ALPHA))
       A4=ALOG((1-1.1*BETA)/(1.5*ALPHA))
       AL=(ALOG(2.))*(1./C0-1./C1)
       AL1=ALOG(2**(C0/C1)-1.)
       P1=1-SQRT(.1584)
       AL2=ALOG(.1584*P1/(.8416*(1-P1)))
       AL3=(ALOG(2.))*(1-(C0/C1))
       AL4=ALOG((1-P1)/.1584)
       AL5=ALOG((1-P41)/(1-P40))
       AL6=ALOG(P41*(1-P40)/(P40*(1-P41)))
        DO 5 I=1,300
        BOUND(I,1,1)=(I*ALOG(C1/C0)+B1)/AL
        BOUND(I,2,1)=(A1-I*ALOG(C0/C1))/AL
        BOUND(I,1,2)=(A2-I*AL3)/AL1
        BOUND(I,2,2)=(B2-I*AL3)/AL1
        BOUND(I,1,3)=(A3-I*AL4)/AL2
        BOUND(I,2,3)=(B3-I*AL4)/AL2
        BOUND(I,1,4)=(A4-I*AL5)/AL6
        BOUND(I,2,4)=(B4-I*AL5)/AL6
    5        CONTINUE
  300  READ(5,6,END=1000) FMED,SHAPE
C     FMED----MEDIAN VALUE TO BE GENERATED
C     SHAPE----SHAPE PARAMETER IN WEIBULL DISTRIBUTION
    6        FORMAT(2F6.3)
       WRITE(6,102)FMED,SHAPE
       KCUNT=1.
       DO 7 I=1,4
       DO 7 J=1,4
    7        DATA(I,J)=0.
C       T1(2,3,4)----TEST STATISTIC FOR TEST CASE A(B,C,D)
C       DCZN1(2,3,4)--TAKES VALUE 1. IF TEST TERMINATED,OTHE
C           RWISE 0.
   10        T1=0.
             T2=0.
             T3=0.
             T4=0.
             N=0.
             DCZN1=0
```

```
          DCZN2=0
          DCZN3=0.
          DCZN4=0
   20 CALL GGUB(ISEED,1,U)
      X=FMED*(ALOG(U)/ALOG(.5))**(1./SHAPE)
      N=N+1
      IF(DCZN1.EQ.1.)GO TO 40
      T1=T1+X
      IF(T1.GT.BOUND(N,1,1).AND.T1.LT.BOUND(N,2,1)) GO TO 40
      TEST(KOUNT,1,1)=N
      DATA(1,1)=DATA(1,1)+N
      IF(T1.LE.BOUND(N,1,1))GO TO 30
      TEST(KOUNT,2,1)=-1.
      GO TO 31
   30 TEST(KOUNT,2,1)=1.
   31 DCZN1=1.
   40 IF(DCZN2.EQ.1.)GO TO 60
      IF(X.LE.CO)T2=T2+1.
      IF(T2.GT.BOUND(N,1,2).AND.T2.LT.BOUND(N,2,2))GO TO 60
      TEST(KOUNT,1,2)=N
      DATA(2,1)=DATA(2,1)+N
      IF(T2.GE.BOUND(N,2,2))GO TO 50
      TEST(KOUNT,2,2)=-1.
      GO TO 51
   50 TEST(KOUNT,2,2)=1.
   51 DCZN2=1.
   60 IF(DCZN3.EQ.1.)GO TO 80
      IF(X.LE.R)T3=T3+1.
      IF(T3.GT.BOUND(N,1,3).AND.T3.LT.BOUND(N,2,3))GO TO 80
      TEST(KOUNT,1,3)=N
      DATA(3,1)=DATA(3,1)+N
      IF(T3.GE.BOUND(N,2,3))GO TO 70
      TEST(KOUNT,2,3)=-1.
      GO TO 71
   70 TEST(KOUNT,2,3)=1.
   71 DCZN3=1.
   80 IF(DCZN4.EQ.1) GO TO 90
      IF(X.LE.R2) T4=T4+1.
      IF(T4.GT.BOUND(N,1,4).AND.T4.LT.BOUND(N,2,4)) GO TO 20
      TEST(KOUNT,1,4)=N
      DATA(4,1)=DATA(4,1)+N
      IF(T4.GE.BOUND(N,2,4)) GO TO 78
      TEST(KOUNT,2,4)=-1.
       GO TO 79
   78 TEST(KOUNT,2,4)=1.
   79 DCZN4=1.
   90 DDD=DCZN1*DCZN2*DCZN3
      IF(DDD.NE.1.)GO TO 20
      KOUNT=KOUNT+1
      IF(KOUNT.LE.NCOUNT)GO TO 10
      DO 100 I=1,4
  100 DATA(I,1)=DATA(I,1)/NCOUNT
      DO 110 I=1,NCOUNT
      DO 110 II=1,4
      DATA(II,2)=(TEST(I,1,II)-DATA(II,1))**2./FLOAT(NCOUNT-
     *1)+DATA(II,2)
      IF(TEST(I,2,II).EQ.1.) DATA(II,3)=DATA(II,3)+1.
  110 CONTINUE
      DO 120 J=1,4
  120 DATA(J,4)=NCOUNT-DATA(J,3)
      WRITE(6,130) ((DATA( I,J),J=1,4),I=1,3)
  130 FORMAT(/,4X,' MEAN',F10.5,'  VAR',F10.5,'  NO ACCEPT '
     *,F5.0,'  NO REJECT ',F5.0)
  101  FORMAT(10F10.5)
  102  FORMAT(5X,' MEDIAN IS',F7.3,'    SHAPE PARAMETER IS',
     *F7.3)
       GO TO 300
 1000  STOP
      END
```

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